ESS5203.03 - Turbulence and Diffusion in the Atmospheric Boundary-Layer: Winter 2020

Spectra and Cospectra

"Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity." Lewis Fry Richardson.

See: Garrat, Section 3.5; and Kaimal and Finnigan, Chapters 2 (all), 3.5, and 7

- 0) Synoptic and Mesoscale ⇒ Spectral Gap ⇒ Energy Containing Range (+ Trend Interference) ⇒ Inertial Subrange ⇒ Dissipation Range (+Aliasing) ⇒ (Noise)
- 1) Spectral Equivalence and Normalization

$$\kappa F(\kappa) = f S(f) = n S(n)$$

$$\frac{f S(f)}{u_*^2 \Phi_{\epsilon}^{2/3}} \quad \text{where} \quad \Phi_{\epsilon} = \frac{kz\epsilon}{u_*^3} \quad \text{or} \quad \frac{f S(f)}{\sigma^2}$$

2) Atmospheric Examples from Garratt Figs. 3.25 and 3.26. Note -2/3 and -4/3 decay rates.



Fig. 3.25 Normalized one-dimensional spectra for (a) the u-component, (b) the w-component and (c) θ in the stable boundary layer; f_m is the frequency of the spectral maximum. Open circles, 0 < z/h < 0.1; triangles 0.1 < z/h < 0.5; solid circles, 0.5 < z/h < 1. Data and curves (representing theoretical expressions) are from Caughey et al. (1979), Journal of Atmospheric Sciences, American Meteorological Society.



Fig. 3.26 Normalized one-dimensional cospectra for (a) u'w' and (b) $w'\theta'$; f_m is the frequency of the spectral maximum. Open circles, 0 < z/h < 0.1; triangles, 0.1 < z/h < 0.5; solid circles, 0.5 < z/h < 1. After Caughey et al. (1979), Journal of Atmospheric Sciences, American Meteorological Society.

3) Spectra within a Canopy (Forest, Vegetation, etc) from Kaimal and Finnigin Figs. 3.11 and 3.12.



FIG. 3.11. Schematic showing energy from wake and waving production feeding directly to the bottom end of the inertial subrange of the energy spectrum, both directly through the action of the mean flow and through the spectral shortcut from large eddies that penetrate the canopy.



FIG. 3.12. Normalized velocity spectra from measurements above the canopy (solid), near the top of the canopy (dashed), and within the canopy (dash-dot) for the (a) *a* component, (b) *v* component, and (c) *w* component.

4) Measuring Spectra:

Averaging period (*T*), Sampling interval (Δt), Sample length ($N = T/\Delta t$), Aliasing: The critical frequency $\left(f_c = \frac{1}{2\Delta t}\right)$.

5) Processing Spectra: Trend removal, Log-scale binning



FIG. 7.10 (a) Spectrum distortion from trends of varying magnitudes (A-D). (b) Sensitivity of low-frequency spectrum shape to three hypothetical detrending procedures (I-III). (Log scales on all axes.)

6) Spikes and Noise