Two identical smokestacks (smokestack A and smokestack B) are separated by 1 km along a North-South line. There is a residential area to the east of the smokestacks. If the height of both smokestacks is 100 m, find the location of maximum concentration at the ground for a 3 m/s westerly wind. Assume linear spread of the plume (i.e. $\sigma_y = I_y x$ and $\sigma_z = I_z x$) with $I_y = 10$ degrees and $I_z = 3$ degrees.

Answer:

Superposition of two plumes. $U = 3$ m/s, $H = 100$ m separation is $L = 500$ m

$I_y = 10^o(\pi/180^o) = 0.174$ and $I_z = 3^o(\pi/180^o) = 0.0524$

From eq. sheet, one Plume is: $C(x,y,z) = \frac{Q}{2\pi U\sigma_y\sigma_z}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right]$

Two plumes offset by $-L$ and $+L$ are added together gives:

$$C(x,y,z) = \frac{Q}{2\pi U I_y I_z x^2}\exp\left[-\frac{1}{2}\left(\frac{z-H}{I_z x}\right)^2\right]\left(\exp\left[-\frac{1}{2}\left(\frac{y-L}{I_y x}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{y+L}{I_y x}\right)^2\right]\right)$$

The maximum (on the ground) will occur at $z = 0$, and $\frac{dC}{dx} = 0$.

Depending on separation of the plumes, the maximum could either occur at $y = \pm L$ or $y = 0$.

Assume from symmetry that the maximum will occur at $y = 0$. Rewrite equation as

$$C = \frac{A}{x^2}2\exp\left[\left(\frac{B}{x}\right)^2\right], \quad \text{where A}= \frac{Q}{2\pi U I_y I_z}, \ B^2 = -\frac{1}{2}\left\{\left(\frac{H}{I_z}\right)^2 + \left(\frac{L}{I_y}\right)^2\right\}$$

$$\frac{dC}{dx} = \frac{d\left\{\frac{2A}{x^2}\exp\left[\left(\frac{B}{x}\right)^2\right]\right\}}{dx}$$

$$\text{Lookup: } \frac{d\left[x^{-2}e^{\left(\frac{a}{x}\right)^2}\right]}{dx} = -\frac{2(x^2+a^2)}{x^5}\exp\left[\left(\frac{a}{x}\right)^2\right]$$

$\frac{dC}{dx} = \frac{-4A(x^2+B^2)}{x^5}\exp\left[\left(\frac{B}{x}\right)^2\right] = 0$, which gives $x = \pm iB$. Sub'ing values gives $x_{y=0} = 2436$ m.

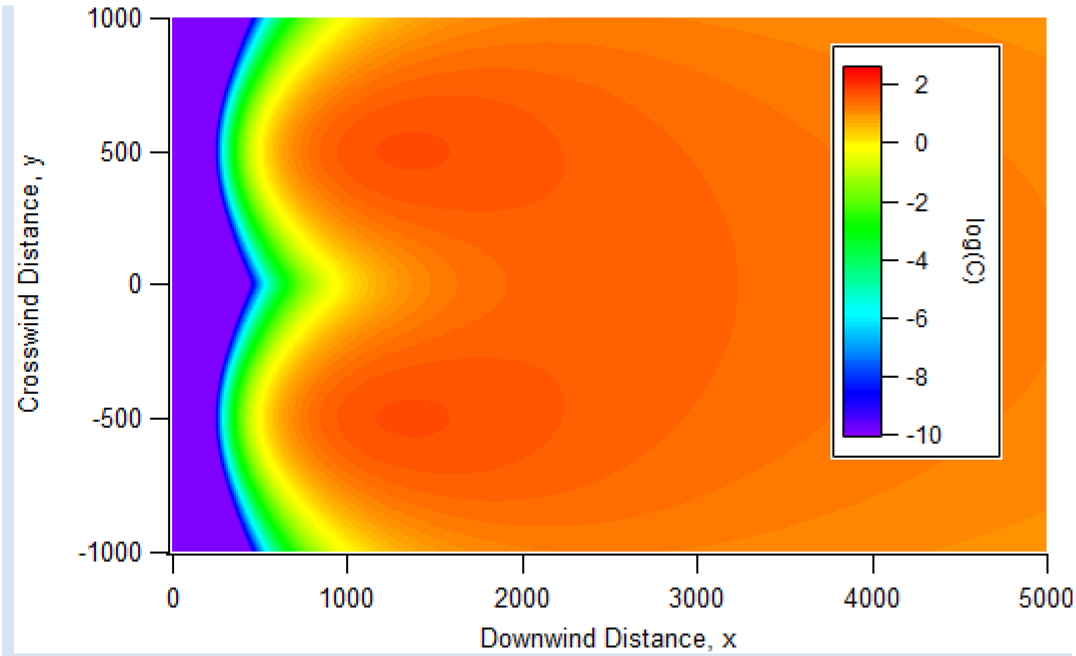
Technically, this should be checked against the $y = \pm L$ values, which is the same except with

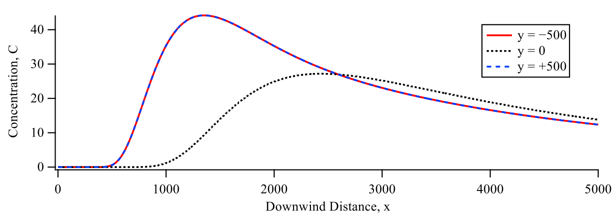
$$C = \frac{A}{x^2}2\exp\left[\left(\frac{B_2}{x}\right)^2\right], \quad \text{where A}= \frac{Q}{2\pi U I_y I_z}, \ B_2^2 = -\frac{1}{2}\left\{\left(\frac{H}{I_z}\right)^2\right\}$$

For $x \pm iB$, this gives $x_{y=L} = 1350$ m.

Since $C\left(x_{y=\pm L}, \pm L, 0\right) = \frac{2A}{x_{y=L}^2}$ and $C\left(x_{y=0}, 0,0\right) = \frac{2A}{x_{y=0}^2}$, the highest concentration must be at $C\left(x_{y=\pm L}, \pm L, 0\right) = \frac{2A}{x_{y=L}^2}$, because $x_{y=L} <$ gives $x_{y=0}$.

Graphical solutions, for illustration:





A tower is set up to measure friction velocity (u_*). The logger records calculations of all wind speed averages ($\overline{u}, \overline{v}, \overline{w}$), variances ($\overline{u^2}, \overline{v^2}, \overline{w^2}$), and covariances ($\overline{uv}, \overline{uw}, \overline{vw}$), for 30 minute periods, but raw, high-frequency measurements (i.e. $u(t)$, etc) are not recorded. Are the campaign it was determined that the tower was tilted by an angle of η. Using the equations for rotation around the y-axis given below, derive an equation for friction velocity accounting for the tower tilt.

$$u_r = u\cos\eta + w\sin\eta$$
$$v_r = v$$
$$w_r = -u\sin\eta + w\cos\eta$$

Answer:

$$u_*^4 = \overline{u_r w_r}^2 + \overline{v_r w_r}^2 = \left(\frac{1}{n}\sum u_r w_r\right)^2 + \left(\frac{1}{n}\sum v_r w_r\right)^2$$

Showing the first term only for example: $\frac{1}{n}\sum u_r w_r = \frac{1}{n}\sum(u\cos\eta + w\sin\eta)(-u\sin\eta + w\cos\eta)$

$$= \frac{1}{n}\sum(-u^2\cos\eta\sin\eta + uw\cos^2\eta - uw\sin^2\eta + w^2\sin\eta\cos\eta)$$
$$= -\overline{u^2}\cos\eta\sin\eta + \overline{uw}(\cos^2\eta - \sin^2\eta) + \overline{w^2}\sin\eta\cos\eta$$