ESS5203.03 - Turbulence and Diffusion in the Atmospheric Boundary-Layer : Winter 2020

Text: J.R.Garratt, The Atmospheric Boundary Layer, 1994. Cambridge, and notes from J.C. Kaimal and J.J. Finnigan, 1994, Atmospheric Boundary-Layer Flows - Oxford. See also http://www.met.rdg.ac.uk/~swrhgnrj/teaching/MTMG49/

1a) General Introduction. Laminar and Turbulent flow, averaging, the atmospheric boundary-layer - diurnal cycle, role of density stratification. G1

1b) Review of governing equations for incompressible flow, continuity, Navier Stokes, equation of state, thermodynamic equation. Vorticity (G2)

Boundary-Layer definitions. Aerodynamics - laminar flow, boundary-layer approximations to N-S equns.

In the absence of any body forces, the Navier - Stokes equations for velocity components, u_i in a viscous fluid can be written, inertial frame of reference. Cartesian tensor notation.

$$\rho\left(\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k}\right) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

while the continuity equation for an incompressible fluid is,

$$\frac{\partial u_j}{\partial x_j} = 0$$

For air $\mu \approx 1.8 \times 10^{-5}$ Nm^{-2}s, $\rho \approx 1.2$ kgm^{-3} then $\nu = \mu/\rho = 1.5 \times 10^{-5}$ m^{2}s^{-1}. (vary with T and p)

ABL or PBL, Surface layer. Roughness sublayer. Flow above a horizontally homogeneous, infinite, flat, plane. Surface roughness z_0.

Geostrophic and gradient wind level. Coriolis force, stratification. Constant Flux layer.

Hodograph and profiles. Ekman spiral (constant eddy viscosity ~ 10m^{2}s^{-1}) - see Garratt p43. more realistic see p46. and profiles below (Weng and Taylor, 2003)

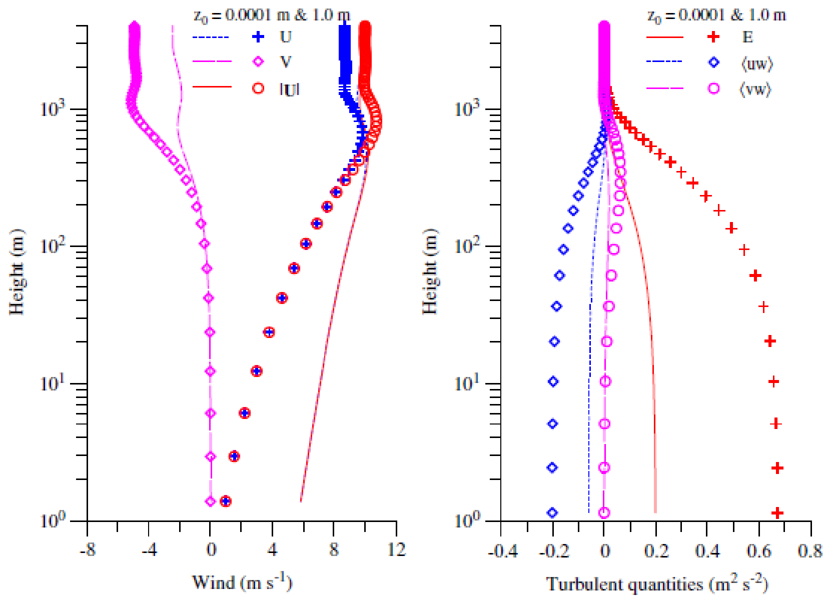

Fig. 2. Initial input profiles of U, V, $|U|$, E, $\langle uw \rangle$ and $\langle vw \rangle$ for Model A runs, from the 1-D PBL model runs of Weng and Taylor (2003) for the given condition of $z_0 = 0.0001$ and 1.0 m, $f = 10^{-4}\,\mathrm{s}^{-1}$, $|\mathbf{U}_g| = 10\,\mathrm{m\,s}^{-1}$, neutral thermal stratification and dry air.

Navier-Stokes and continuity equations. Role of molecular viscosity. Laminar and turbulent flows. Range of scales mm to km. Thermodynamics, potential temperature. Turbulent fluxes, momentum, heat, water vapour, etc. Surface energy budget.

U = U + u'. **Averages!** Reynolds rules. $u_*^2 = -u'w'$ (with x-axis aligned with surface wind). The friction velocity.

PBL depth, of order 1 km. Stresses, fluxes drop by about 10% over lowest 100m, consider them approximately constant up to ? 50m, 100m .....

Buckingham's pi theorem, see https://en.wikipedia.org/wiki/Buckingham_π_theorem

Statement [edit]

More formally, the number of dimensionless terms that can be formed, p, is equal to the nullity of the dimensional matrix, and k is the rank. For experimental purposes, different systems that share the same description in terms of these dimensionless numbers are equivalent.

In mathematical terms, if we have a physically meaningful equation such as

$$f(q_1, q_2, \ldots, q_n) = 0,$$

where the q_i are the n physical variables, and they are expressed in terms of k independent physical units, then the above equation can be restated as

$$F(\pi_1, \pi_2, \ldots, \pi_p) = 0,$$

where the π_i are dimensionless parameters constructed from the q_i by $p = n - k$ dimensionless equations — the so-called *Pi groups* — of the form

$$\pi_i = q_1^{a_1}\, q_2^{a_2} \cdots q_n^{a_n},$$

where the exponents a_i are rational numbers (they can always be taken to be integers: just raise it to a power to clear denominators).

Used in several places in boundary-layer turbulence. A key application is:

Constant stress layer, unidirectional flow, distance from flat, uniform, rough surface, z, Neutral stratification.

Considser $dU/dz = f(u_*, z, ???)$, Assume far enough from surface that details of the roughness elements are not important.

Dimensional considerations, $[dU/dz] = T^{-1}$, $[u_*] = LT^{-1}$, $[z] = L$.

So 3 variables, 2 dimensions, one $\pi_1 = zdU/dz/u_*$. so must have π_1 = constant, $(1/k)$ - by convention.

$$dU/dz = u_*/kz.$$

Has a problem at z = 0 stay away from roughness elements. Suppose we integrate the equation?

$$U = (u_*/k) \ln z + \text{Constant}$$

Suppose $U = 0$ at $z = z_0$.- the roughness length - $U = (u_*/k) \ln (z/z_0)$. ? Observations.

FLOW OVER TOPOGRAPHY: IMPACTS OF THE ASKERVEIN STUDY 307

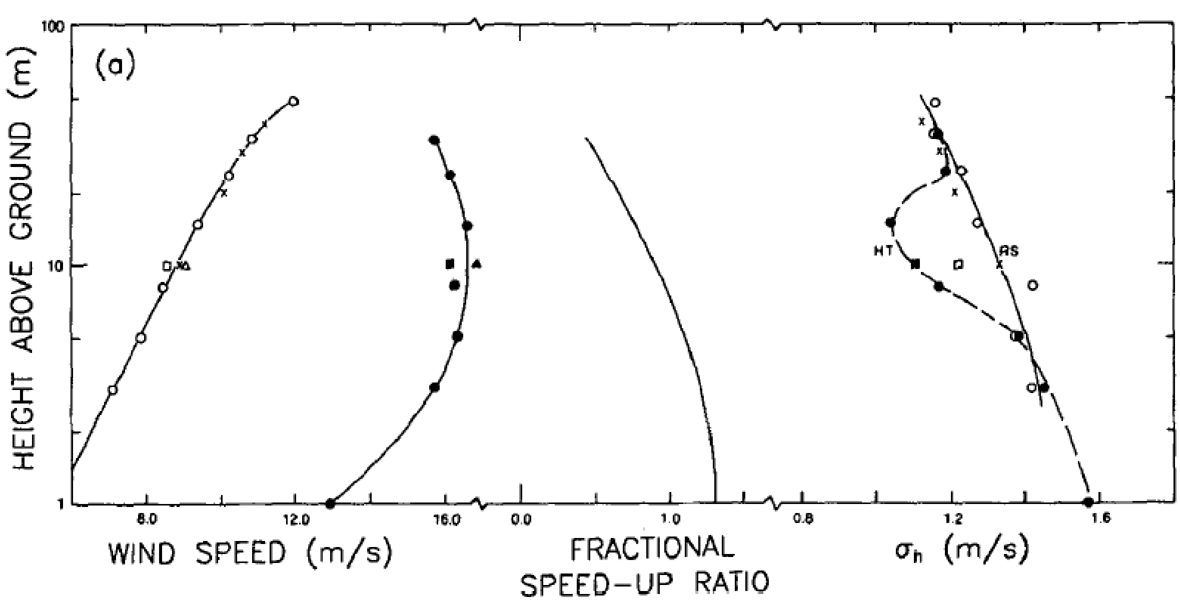

Figure 8a. Mean wind speed, fractional speed-up ratio and σ_h profiles at the reference site, RS (open symbols and ×) and the hilltop, HT (closed symbols) for the 1983 Askervein experiment, Run TU03-B (upwind wind direction ≈ 210°). Source: Mickle *et al.* (1988).

See also http://www.yorku.ca/pat/research/Askervein/index.html

Modified form for convenient model application, $U = (u_*/k) \ln (1+z/z_0)$. Then $U = 0$ at $z = 0$.

Typical z_0 values, https://en.wikipedia.org/wiki/Roughness_length

Terrain description	(*m*)
Open sea, Fetch at least 5 km	0.0002
Mud flats, snow; no vegetation, no obstacles	0.005
Open flat terrain; grass, few isolated obstacles	0.03
Low crops; occasional large obstacles, $x/H > 20$	0.10
High crops; scattered obstacles, $15 < x/H < 20$	0.25
parkland, bushes; numerous obstacles, $x/H \approx 10$	0.5
Regular large obstacle coverage (suburb, forest)	1.0
City centre with high- and low-rise buildings	≥ 2

x is fetch over surface. Footprint issues. Note that ln z is the relevant quantity. Small variations in z0 not critical, variations by factor 10 or more affect flow, e.g water - land.

Rough guide, approximately 1/30- 1/10 of size of roughness elements. Sand grain roughness, d/30.

Navier-Stokes Equations.

Cartesian co-ordinates on an f-plane, Boussinesq Approximation

$$Du/Dt = -(1/\rho)\partial p/\partial x + fv + \upsilon(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 + \partial^2 u/\partial z^2)$$

$$Dv/Dt = -(1/\rho)\partial p/\partial y - fu + \upsilon(\partial^2 v/\partial x^2 + \partial^2 v/\partial y^2 + \partial^2 v/\partial z^2)$$

$$Dw/Dt = -(1/\rho_0)\partial p_p/\partial z -(\rho_p/\rho_0)\, g + \upsilon(\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2 + \partial^2 w/\partial z^2)$$

p_p and ρ_p pressures and densities as perturbations from a background hydrostatic state.

Noting that, $Du/Dt = \partial u/\partial t + u\partial u/\partial x + v\partial u/\partial y + w\partial u/\partial z$ etc.

Continuity $\partial\rho/\partial t + \nabla(\rho\mathbf{U}) = 0$ or, equivalently, $D\rho/Dt + \rho\nabla(\mathbf{U}) = 0$

Incompressible fluid, $D\rho/Dt = 0$ so $\nabla(\mathbf{U}) = 0$.

Mean + turbulence perturbation $u = U + u'$ etc, $p = p_0 + p_b + p'$

Means can be 1-D, 2-D, 3-D, steady state or time dependent. Perturbations are always 3D and time varying.

Reynolds number $R = UL/\upsilon$. High R means molecular viscous termd neglected.

Reynolds Averaged Navier Stokes, RANS equations, b-layer approx.

$$\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z = -(1/\rho)\partial p_b/\partial x + fV - \partial\langle u'w'\rangle/\partial z$$

$$\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z = -(1/\rho)\partial p_b/\partial y - fU - \partial\langle v'w'\rangle/\partial z$$

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0 \ : \ \text{Note} \quad \partial u'/\partial x + \partial v'/\partial y + \partial w'/\partial z = 0 \quad \text{as well}$$

Assume $p_b(x,y)$ specified and replaced by Geostrophic wind term.

$$-(1/\rho)\partial p/\partial y - fU_g = 0, \quad -(1/\rho)\partial p/\partial x + fV_g = 0$$

1-D case:

$$\partial U/\partial t = + f(V-V_g) - \partial\langle u'w'\rangle/\partial z \ ; \ \partial V/\partial t = -f(U - U_g) - \partial\langle v'w'\rangle/\partial z$$

Since $\partial U/\partial x + \partial V/\partial y = 0$, W = const = 0 (flat ground). Two equations 4 unknowns

Eddy Viscosity

$$\tau_x/\rho = \langle -u'w'\rangle = K\,\partial U/\partial z \ : \ \tau_y/\rho = \langle -v'w'\rangle = K\,\partial V/\partial z$$

A LOCAL gradient hypothesis - non-local closure schemes?

K = constant ? Mixing length $K = u_* l(z)$ or $u_* kz$ (from surface layer, log profile)

Ekman layer solution, constant K, steady state. Let $W = U + iV$, $V_g = 0$

leads to $U = U_g[1-e^{-\alpha z}\cos(\alpha z)]$; $V = (f/|f|)U_g e^{-\alpha z}\sin(\alpha z)$ where $\alpha^2 = |f|/(2K)$.

Other closures. K = velocity scale x length scale. May depend on stability ($d\theta_0/dz$).

Velocity scales, u_* or $E = ((u'^2 + v'^2 + w'^2)/2)^{1/2}$, TKE equation needed. ($E^2$ = TKE per unit mass) TKE equation includes ε - dissipation rate. Usually αE,

Length scales? kz, $k(z+z_0)$, $1/l = 1/k(z+z_0) + 1/\lambda$. Blackadar , $\lambda = 0.00027|\mathbf{U}_g|/f$
cf. Taylor (1969).

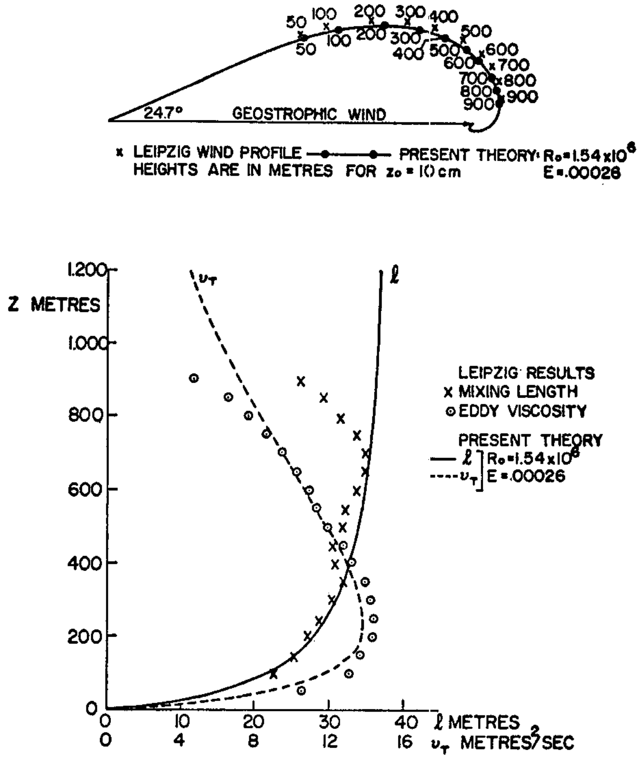

FIG. 2. Comparison of theoretical results with the "Leipzig wind profile."

With stratification can use (Delage, 1974)

$$\frac{1}{\ell_m} = \frac{1}{\kappa(z+z_0)} + \frac{1}{\ell_0} + \frac{\beta_c}{\kappa L_O}$$

TKE equation derivation. Full equn for $\partial u_i/\partial t$, subtract $\partial U_i/\partial t$ equation and get equation for $\partial u_i'/\partial t$, Multiply by u_j', add $u_i'\partial u_j'/\partial t$, average to get equn for $\partial \langle u_j' u_i' \rangle/\partial t$, let i =j and apply summation. Include viscous terms which lead to dissipation.

For stratification also need thermodynamic equation, for potential temperature, θ

$$\partial\theta/\partial t + u\partial\theta/\partial x + v\partial\theta/\partial y + w\partial\theta/\partial z \approx 0$$ but there may be heat sources.

Mean + perturbation and average, and the assumption that $\langle w'\theta' \rangle = -K_h \partial\theta/\partial z$.