ESS5203.03 - Turbulence and Diffusion in the ABL : Winter 2020 - Notes 2b

Text: J.R.Garratt, The Atmospheric Boundary Layer, 1994. Cambridge, Chapter 8

Boundary-Layer Modelling issues.

Modelling within large scale models, modelling of the boundary-layer. Surface layer or **Planetary BL**?

1-D steady state, **1-D time dependent - diurnal cycles**, 2-D, 3-D steady state.

Boundary conditions, lower and top. How to treat surface? Coupled to soil (or ocean or ice) model.

CLASS

Slab, RANS local or non-local closure, 1st, 1.5, 2nd order closures, LES.

Where to start? Garratt puts stress on surface boundary. Energy budget issues.

1-D Diurnal cycle (Weng and Taylor, 2003, Boundary-Layer Meteorology)

ON MODELLING THE ONE-DIMENSIONAL ATMOSPHERIC BOUNDARY LAYER

In an idealised, horizontally homogeneous ABL and in the absence of radiative flux divergence and moisture, the Reynolds averaged equations describing the dynamics of the ABL can be written as

$$\frac{\partial U}{\partial t} = f(V - V_g) - \frac{\partial \langle uw \rangle}{\partial z}, \tag{1}$$

$$\frac{\partial V}{\partial t} = f(U_g - U) - \frac{\partial \langle vw \rangle}{\partial z}, \tag{2}$$

$$\frac{\partial \Theta}{\partial t} = -\frac{\partial \langle w\theta \rangle}{\partial z}, \tag{3}$$

In this paper we study some of the closure schemes commonly used in ABL modelling. They are all $1\frac{1}{2}$-order schemes in which the equations for the turbulent kinetic energy (TKE) and a turbulence length scale are used. The turbulent length scale equation can either be diagnostic ($E - \ell$, $E - \epsilon - \ell$ and $q^2\ell$ Model I) or prognostic ($E - \epsilon$, or its modification and $q^2\ell$ Model II).

$$- \langle uw \rangle = K_m \frac{\partial U}{\partial z}\,, \tag{4}$$

$$- \langle vw \rangle = K_m \frac{\partial V}{\partial z}\,, \tag{5}$$

$$- \langle w\theta \rangle = K_h \frac{\partial \Theta}{\partial z}\,, \tag{6}$$

$$K_m = \ell_m \, (\alpha E)^{1/2}\,, \tag{7}$$

$$K_h = \ell_m \, (\alpha E)^{1/2} / \, Pr\,, \tag{8}$$

where ℓ_m is a turbulent mixing length, E is the turbulent kinetic energy (TKE), the constant α is the ratio of the surface shear stress to TKE and Pr is the turbulent Prandtl number (defined as $Pr = K_m/K_h$, the value of 0.74 is used here). One approach to model ℓ_m is simply to formulate a diagnostic equation for the length scale that relates it to the distance from the surface and the stability of the atmosphere; another uses the prognostic equation for the dissipation rate of TKE, ϵ. One can then formulate a length scale from E and ϵ, i.e.,

$$\ell_d = (\alpha E)^{3/2} / \, \epsilon\,, \tag{9}$$

Both approaches need an equation for the TKE, which, in one-dimensional form, is

$$\frac{\partial E}{\partial t} = P_s + P_b - \epsilon + \frac{\partial}{\partial z}\left(K_m \frac{\partial E}{\partial z} \right)\,, \tag{12}$$

where P_s and P_b are TKE production terms by the shear and buoyancy respectively

$$P_s \equiv - \langle uw \rangle \frac{\partial U}{\partial z} - \langle vw \rangle \frac{\partial V}{\partial z} \qquad \text{with} \qquad P_b \equiv \beta g \, \langle w\theta \rangle \,.$$

Here β is the coefficient of thermal expansion, g is the acceleration due to gravity.

E-*l*, E-ε, q^2-*l* closures are 1.5 order or (low) 2nd order closures. Can also formulate full 2nd order closure.

E-ε needs an equation for ε.

(11). In the $E - \epsilon$ closure scheme the rate of dissipation of TKE, ϵ, is governed by the prognostic equation,

$$\frac{\partial \epsilon}{\partial t} = \frac{\epsilon}{E} C_{\epsilon 1} \left(P_s + P_b\right) - C_{\epsilon 2} \frac{\epsilon^2}{E} + C_\epsilon \frac{\partial}{\partial z} \left(K_m \frac{\partial \epsilon}{\partial z} \right) , \tag{19}$$

where the three terms on the RHS are loosely labelled as the production, dissipation or destruction and diffusion of ϵ respectively. The often used constants are $C_{\epsilon 1} = 1.44$, $C_{\epsilon 2} = 1.92$ and $C_\epsilon = \alpha(C_{\epsilon 2} - C_{\epsilon 1})/\kappa^2$.

Based on a creative imagination? Seems to work well and is widely used in engineering applications but needs some adjustment in PBL. More creative imagination in q^2-l equation, Mellor, Yamada

$$\frac{\partial q^2 \ell}{\partial t} = E_1 \ell \Big(P_s + P_b \Big) - \frac{q^3}{B_1} \left[1 + E_2 \left(\frac{\ell}{\ell_R} \right)^2 \right] + \frac{\partial}{\partial z} \left(\ell q S_\ell \frac{\partial q^2 \ell}{\partial z} \right) , \tag{25}$$

As in other studies, the model uses a stretched vertical coordinate to ensure sufficient resolution near the surface and to resolve strong gradients. We set

$$Z = \ln \frac{z + z_0}{z_0} + \frac{z}{b_0} , \tag{33}$$

where b_0 is a constant (67.5 m is used in these calculations). Equations are transformed into the new coordinate system before they are discretized into their finite difference equivalents. Flow variables are stored on a staggered grid, where mean variables (U, V and T) are at layer midpoints and turbulent quantities (E and turbulence fluxes) at layer lower boundary levels and z_t (the top of the computation domain). The numerical scheme employed for time integration is Crank–Nicolson. The resulting set of difference equations is solved using a block **LU** factorization algorithm (Karpik, 1988).

The surface boundary conditions used are a non-slip condition for velocity ($U = V = 0$), a specified time dependent temperature or heat flux if required, and the assumption that production balances the dissipation of TKE ($P = \epsilon$). At the upper boundary, we specify $(U, V) = (U_g, V_g)$, $\Theta = \Theta_g$ (constant) and set the vertical derivatives of TKE, ϵ, shear stresses and other turbulent fluxes to zero.

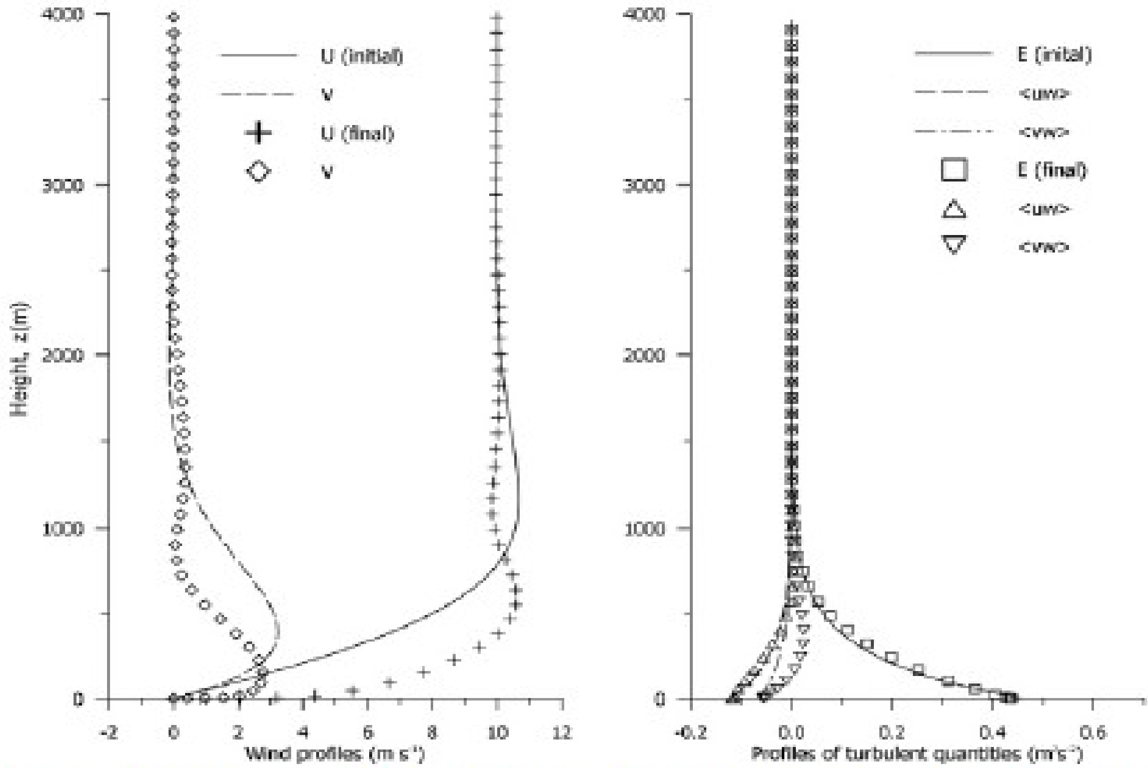


Figure 1. Initial profiles of wind (U, V), TKE (E) and shear stress $(\langle uw\rangle, \langle vw\rangle)$ and results from $E-\ell$ closure under neutral stratification.

Initial profiles - neutral stratification E-l above E-ε below

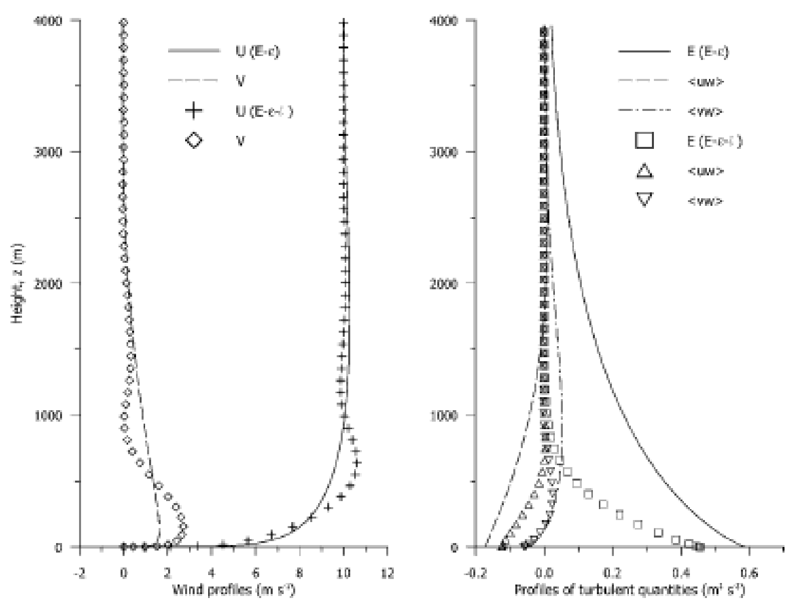
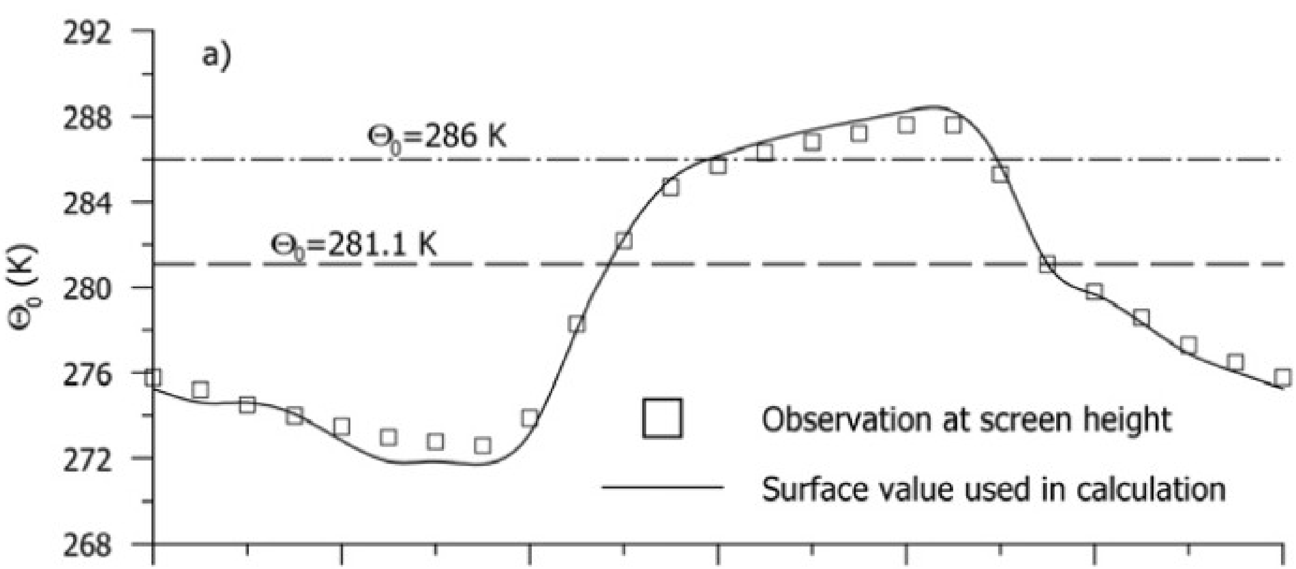


Figure 2. Vertical profiles of wind (U, V), TKE (E) and shear stress $(\langle uw\rangle, \langle vw\rangle)$ from standard $E-\epsilon$ and $E-\epsilon-\ell$ closures under neutral stratification.

4.3. Diurnal cycle

Our diurnal cycle studies are based on running the models by specifying the diurnal variation of the surface potential temperature (Θ_0) or the surface heat flux ($\langle w\theta\rangle_0$). Several forms were tried including the sinusoidal variations of Θ_0 and an empirical formulation of $\langle w\theta\rangle_0$ (Stull, 1988, Section 7.3). Here, to construct a typical diurnal variation of Θ_0, we use the measured screen height ($z = 2$ m) temperature from Day 33 of the Wangara Experiment (Clarke et al., 1971). We take their measurements of hourly potential temperature, Θ_i ($i = 0, 1, \ldots, 23$ representing different hours), at screen height and then set the surface potential temperature, Θ_0 at 1900 hours, equal to Θ_{19} and estimate Θ_0 at other hours by $\Theta_0 = \Theta_{19} + 1.1 \times (\Theta_i - \Theta_{19})$, where $i = 0, 1, \ldots, 23$ (The factor 1.1 is an attempt to extrapolate the screen height temperature to the surface). A cubic spline interpolation was used for Θ_0 at times between hours, see Figure 14a. The initial potential temperature profile for all the model runs is given by $\Theta(z, 0) = \Theta_{19} = 281.1$ K for $0 \le z \le 1000$ m, and $\Theta(z, 0) = 281.1 + 0.0035\,(z - 1000)$ for $z > 1000$ m. The initial velocity field is a steady state, neutral ABL with $(U_g, V_g) = (10, 0)$ m s^{-1}, $f = 10^{-4}$ s^{-1} and $z_0 = 0.1$ m. Thereafter the surface potential temperature is allowed to vary according to Figure 14a with the calculation starting at 1900 local time (LT).



So we cover stable and unstable cases. Initial condition is steady state neutrally-stratified solution. Integrate for 10 days repeating diurnal cycle, Upper boundary had Ug = 10 ms^{-1}, Vg = 0, θ = 271.1K.

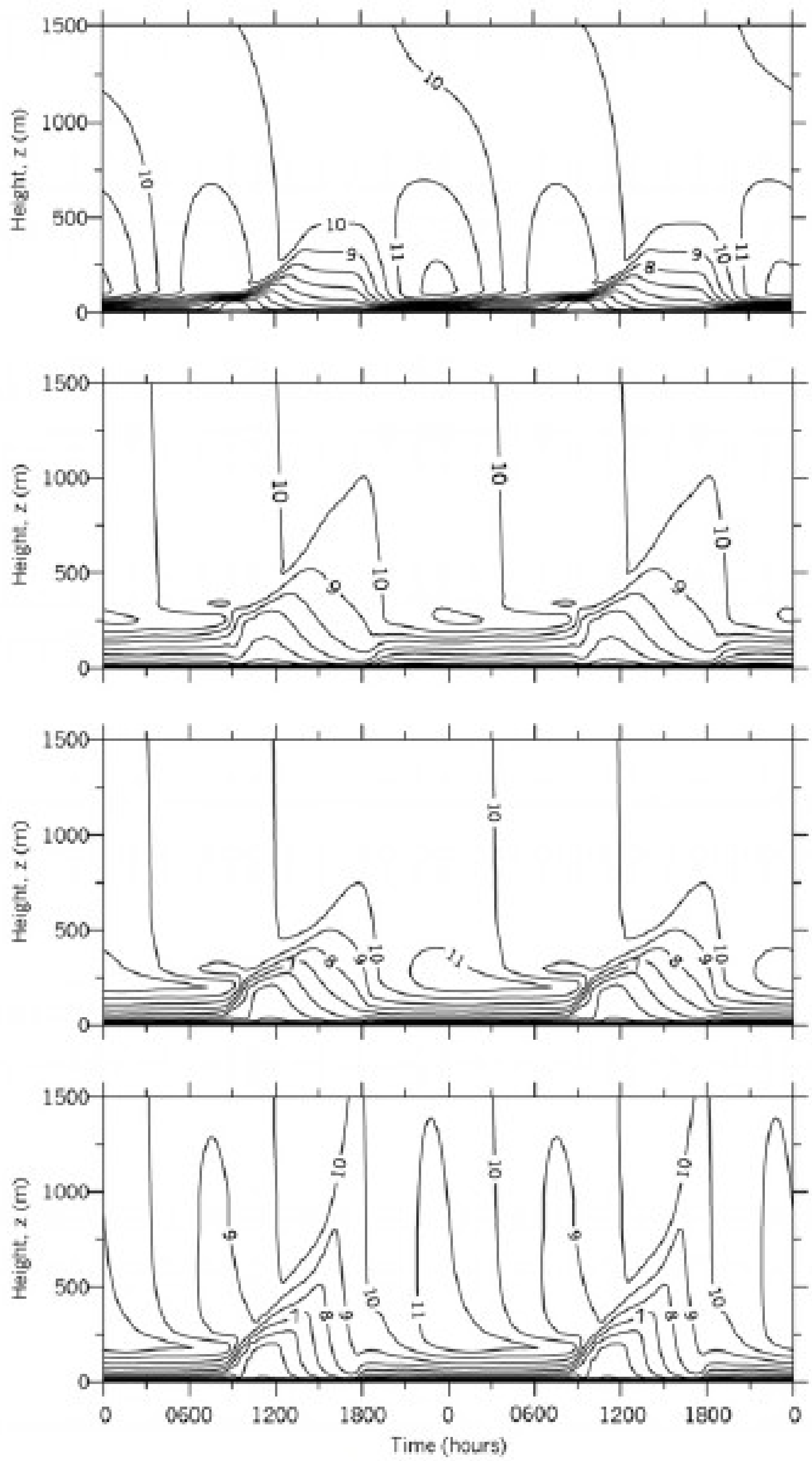


Figure 15. Mean wind component, U (m s^{-1}), from different turbulence closures. From top to bottom the closures used are $E-\ell$ (D74), $E-\epsilon$ (the standard), $q^2\ell$ Models I and II.

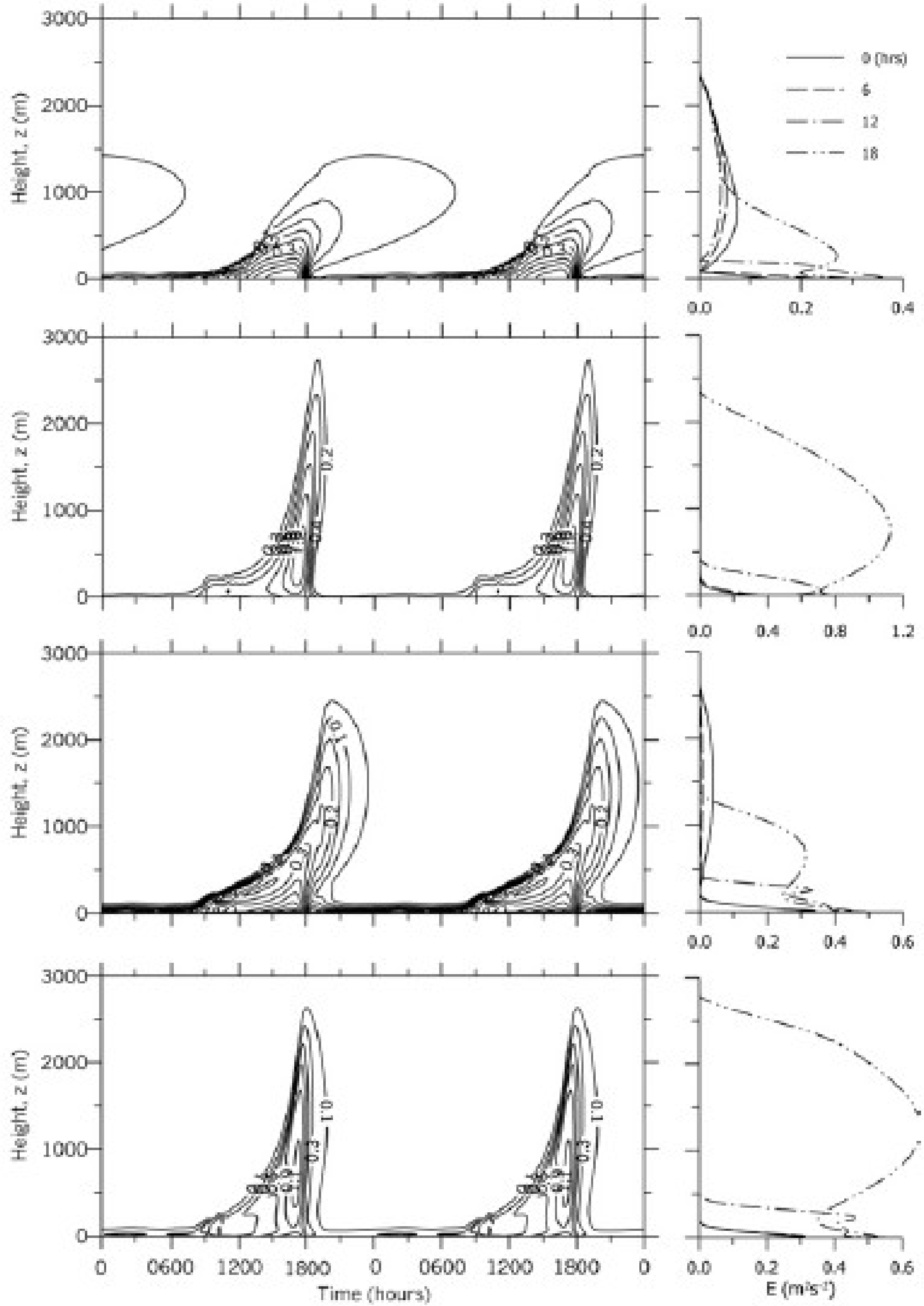


Figure 17. The TKE, E (m^2 s^{-2}), from different turbulence closures. From top to bottom the closures used are $E - \ell$ (D74), $E - \epsilon$ (the standard), $q^2\ell$ Models I and II. Note the different contour intervals and different x-axis scales in profile plots.

5. Conclusions

Several different $1\frac{1}{2}$-order turbulence closure models are used to simulate the atmospheric boundary layer. The eddy viscosity concept is employed to model the turbulent fluxes. All models use the turbulent kinetic energy equation together with either a diagnostic expression or a prognostic equation to represent the turbulent length scale. The effects of stability are realized via the turbulent length scale and the TKE in $E - \ell$ closure, the TKE and the dissipation rate of TKE in $E - \epsilon$ closure and its modified versions and the turbulent length scale, the TKE and a stability function in $q^2\ell$ Models I and II.

The turbulence length scale plays a very important role in any turbulence closure models. In the simple situations studied here, the models with TKE and a diagnostic equation for ℓ are quite good, while the closures with TKE and a prognostic equation for ℓ do not guarantee success although these models do carry more physical processes.

Full second-order closure scheme models, which can predict all components of the Reynolds stress, have significant advantages for some flow situations, e.g., flow over hills. Ayotte and Taylor (1995) used the LRR second-order scheme for their neutral, linear ABL model of flow over topography but limited the time integration of the background flow calculation to 4 hr in order to avoid having too deep a boundary layer. The problem with the ϵ- or $q^2\ell$-equation, and prediction of a very deep ABL and large values for the length scale in this flow situation, need to be resolved. However a model including prognostic equations for the shear stress is required for flows that are rapidly distorted by topography.

Boundary-Layer Meteorology **107:** 371–400, 2003.

Computational issues,

Finite difference grid, explicit or implicit, near singularity at ground, z-transformation and wall layers.

Better treatment of the surface boundary condition, for potential temperature and mixing ratio. Coupling with soil layer.