SS5203.03 - Turbulence and Diffusion - 2020
PROBLEM SET #1, due Jan 23.

1) Review Cartesian Tensors δ_{ij} and ε_{ijk}, scalar ($\bullet$) and vector ($\mathbf{x}$) products, and the ε-δ relationship, $\varepsilon_{ijk}\varepsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$. The Wikipedia site is one place to look, https://en.wikipedia.org/wiki/Cartesian_tensor

   a) Show that curl(grad φ) = 0

2) Review the Ekman spiral solutions for a constant eddy viscosity (Garratt section 3.1.2) and show that the surface cross isobar angle is 45°. Plot hodographs for $\underline{U_g}$ = (10,0) ms^{-1} , f = $10^{-4}s^{-1}$ with both K = 10 m^2s^{-1} and K = 1 m^2s^{-1} . Briefly explain how they are different.

3) Suppose you measure half hour average wind speeds from anemometers on a 10m tower under cloudy skies in homogeneous terrain with a low grass surface cover, and obtain U(10m) = 14.0 m/s, U(4m) = 12.2 m/s and U(2m) = 10.5 m/s. Obtain best estimates for z_0 and u_*, assuming k = 0.4, noting that you have 2 unknowns and 3 equations. Why would 2 levels be easier to work from?

4) Starting from the Navier-Stokes (with the Boussinesq approximation) and continuity equations, derive the equations for the time rate of change of the Reynolds stresses, $\langle u_i u_j \rangle$. Set i = j to obtain the turbulent kinetic energy (TKE) equation, (Garratt, Equn 2.72) and give an explanation of the terms in the Equation (shear production, buoyancy production, viscous dissipation).