SS5203.03 - Turbulence and Diffusion - 2020
PROBLEM SET #2, due Feb 14, 2020.

1) Consider an idealised situation of turbulent boundary layer flow over a step change in surface roughness. Assume that a constant flux, neutrally stratified boundary layer above the region $x < 0$ is in a steady state over a horizontally homogeneous flat upstream surface with a roughness length (z_{01}) of 0.001m (maybe a snow surface) and has $U = U_0(z)$ in the positive x direction. For $x > 0$ the roughness length increases to $z_{02} = 0.1$m. Assume that the wind speed at $z = 50$m is fixed at $U = 10$ ms^{-1}. Ignore Coriolis force effects. Assuming a simple E-l closure with $l = k(z+z_{02})$ write down the governing equations, plus initial and boundary conditions that could be used to model the time evolution of the flow.

2) For flow over a step change in roughness there is a concept of an "internal boundary layer" with a simple (implicit equation) model (see Garratt p111, but modified so that $\delta = 0$ at $x = 0$) estimating the depth, $\delta(x)$, as satisfying

$$[(\delta+z_{02})/z_{02}][\ln((\delta+z_{02})/z_{02}) - 1] + 1 = 0.5\, x/z_{02}.$$

Use $z_{02} = 0.1$m. Solve this implicit equation (numerically/iteratively) to obtain δ as a function of x for $0<x<500$m and plot the curve.

Relate this to Question 1 to estimate how far it would take (X) for the change in surface roughness to impact the wind speed at $z = 50$m. Can use simple linear iteration. Useful to let $Z = (\delta+z_{02})/z_{02}$.

3) Write a computer program using finite differences to solve the governing equations established in Question 1 (maybe after a transformation of the z coordinate to $\varsigma = \ln(z + z_{02})$) and preferably with an implicit scheme) with the appropriate initial and boundary conditions, and run it for $0 < z < 100$m and $0 < x < 2X$ where X is your estimate from Qu 2. You could make the approximation $W = 0$ to simplify the method. Plot wind speed profiles at several intermediate times.
