ESS 5203: Turbulence and Diffusion in the Atmosphere

Assignment #3 Due March 16

1) Winds have been blowing strongly from due W over Lake Erie for several days. Wind stress on the surface is 0.2 Nm^{-2}. Ignore any coastal effects, Assume horizontal homogeneity, no surface slope and a well-mixed surface layer with a very strong thermocline at a depth of 5m. Assume $f = 10^{-4}\ s^{-1}$. In the simplest, idealized case with no mixed layer deepening, what currents would you expect in that upper mixed layer? Suppose that the wind suddenly stops blowing. What happens to the currents? Explain the idealizations that you make.

I posted a suggestion to look at Paduan et al (1989) and also posted the suggestion,

Treat the upper mixed layer as a single slab with a mean velocity (U,V) Take the x axis pointing due E. Assume no stress at the bottom of the mixed layer (strong thermocline).

Then balance of forces (surface stress and Coriolis force) will lead to geostrophic balance

$-\rho f V = \tau_s/d$ and $U = 0$. d is mixed layer depth

Gives rather a high current (40 cm/s towards the S) so I should have made d larger, 20m would have been better or reduce the stress.
This is the solution of Paduan Equn (1) with steady state dQ/dt = 0.

Now put dQ/dt back and set the wind stress to 0. ....

Paduan Equation 1 is within

3. Wind-Forced Slab Model

We compute the inertial currents $\mathbf{Q} = u + iv$ driven by the surface wind stress $\rho\mathbf{Y} = \tau^x + i\tau^y$ according to a simple slab model in which the stress is taken up uniformly over a constant depth h:

$$\frac{d\mathbf{Q}}{dt} + \Omega\mathbf{Q} = \frac{\mathbf{Y}}{h} \tag{1}$$

where $\Omega = r + if$. Ad hoc Raleigh frictional decay is included with an e-folding time of r^{-1}. The solution is

$$\mathbf{Q} = \mathbf{Q}_0 e^{-\Omega t} + \int_0^t \frac{\mathbf{Y}}{h} e^{-\Omega(t-t')}\, dt' \tag{2}$$

where $\mathbf{Q}_0$ is the initial current [*Phillips*, 1966]. This simple model was used successfully by *Pollard and Millard* [1970] and has since been used many times to predict wind-driven currents within the mixed layer [*Kundu*, 1976*a*; *Käse and Olbers*, 1980; *Pollard*, 1980; *D'Asaro*, 1985*a*, *b*].

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One can look at Equn (1) in terms of the single complex variable Q or as two equations in u and v. Either way leads to 40 cm/s towards the South.

Now if the stress is turned off at some time, say at t =0 with $Q_0 = (U_0+iV_0) = -40i$ we have the equation,

$dQ/dt + \Omega Q = 0$, where $\Omega = if$ when there is no frictional decay and f is the Coriolis parameter.

In complex variables the solution is $Q = Q_0\exp(-\Omega t)$. In terms of components we have

$$\begin{aligned} u + iv = iV_0 \exp(-ift) &= iV_0(\cos(-ft)+i\sin(-ft)) \\ &= V_0(-\sin(-ft) + i\cos(-ft)) = 40\,(\sin(-ft) - i\cos(-ft)) \end{aligned}$$

The easiest way to picture this is in terms of speed and direction.

$$\text{Speed,}\quad |Q| = (u^2+v^2)^{1/2} = 40\,(\cos^2 ft + \sin^2 ft)^{1/2} = 40 \text{ m/s}$$

For direction my thought would be to express the velocity solution as

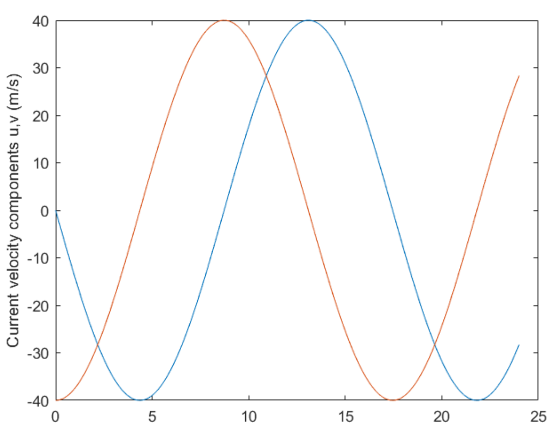
$$u + iv = 40\,(\cos F(t) + i\sin(F(t)) \text{ ------ (A)},$$

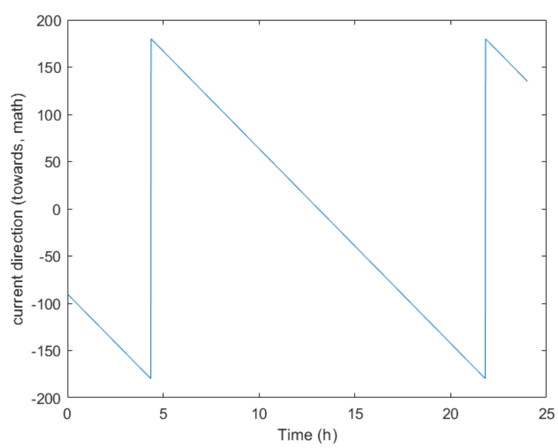
and find an appropriate F(t) in the form $-\Omega t + \varphi$ where φ is a phase shift

We already have $u + iv = -40\,(-\sin(-ft) + i\cos(-ft))$; at t=0, u+iv = -40i so in terms of (A)

we need $\cos(F(0)) = 0$ and $\sin(F(0)) = -1$. We can achieve this if $F(t) = -ft + 3\pi/2$.

So the solution that I propose is $u + iv = 40\,(\cos(-ft + 3\pi/2) + i\sin(-ft + 3\pi/2))$. Plots below.





U (blue) and V (red) current components plus direction (math axes and towards so initial current (0, -40)m/s. Code was simply,

```
%Inertial; T=0:60:24*3600;TH=T/3600;V0=40; f=10^-4;ts = 24*60 + 1; for in = 1:ts; t= T(in);
u(in)= V0*cos(-f*t+1.5*pi); v(in)= V0*sin(-f*t + 1.5*pi);  dir(in) = (180/pi)*atan2(v(in),u(in));
end; figure; plot(TH,u);hold on; plot(TH,v); figure; plot(TH,dir);
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Without using complex variables, and starting from, …. if the stress is turned off at some time, say at t =0 with $Q_0 = (U_0+iV_0) = -40i$ we have the equation,

$dQ/dt + \Omega Q = 0$, where $\Omega = if$ when there is no frictional decay and f is the Coriolis parameter.

i.e, $d(u+iv)/dt + if(u+iv) =0$ which split into real and imaginary parts gives

$du/dt - fv = 0$: $dv/dt + fu = 0$; Eliminating v by differentiating gives,

$d^2u/dt^2 - f dv/dt = 0$, and so $d^2u/dt^2 + f^2u =0$. Soln $u = A \cos ft + B \sin ft$

at t = 0 we have u = 0, v = -40 ms^{-1} so A = 0 and $fv = du/dt = Bf = -40f$ ms^{-2}. So B = -40 ms^{-1}.

Thus solution is $u = -40 \sin ft$: $v = -40 \cos ft$, OR $u = 40 \sin -ft$: $v = -40 \cos -ft$ as before.
