The condensation of water vapour into droplets and the formation of fog in the Earth's atmospheric boundary layer involves a complex balance between horizontal advection and vertical turbulent mixing of heat and water vapour, cloud microphysical processes involving the numbers and size of available condensation nuclei and radiative transfers of heat, plus the impact of water droplets, and sometimes ice crystals, on visibility. It is a phenomenon which has been studied for many years in a variety of contexts. Over the waters offshore from Newfoundland a key factor is the advection of moist air from over warm gulf stream waters to colder Labrador current water - an internal boundary-layer problem. Some basic properties can be learned from a steady state 2-D (x-z) model.

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Upstream conditions: a 1-D time dependent model.

Surface cooling, $dT_s(t)/dt < 0$, so $q_s(T_s)$ also decreasing. Water surface is a water droplet sink, $Q_l = 0$, and potential source or sink of $Q_v$. Settling velocity (Nakanishi, 2000 proposes $V_T = 10^6(Q_l/N_0)^{2/3} \text{ms}^{-1}$ - an empirical equation! with $N_0$ fixed at $10^8\text{m}^{-3}$). Typical values, $Q_l = 10^{-4}$, $V_T = 0.01\text{ms}^{-1}$.
Models we have learnt from.


Nakanishi, 2000, Large eddy simulation of radiation fog, Boundary-Layer Meteorology 94: 461–493,


Based on Weng and Taylor (2003), in the Planetary Boundary Layer, 1D PBL model equations are

\[
\frac{\partial U}{\partial t} = - \frac{\partial \langle uw \rangle}{\partial z} + f(V - V_g), \tag{1}
\]

\[
\frac{\partial V}{\partial t} = - \frac{\partial \langle vw \rangle}{\partial z} - f(U - U_g), \tag{2}
\]

\[
\frac{\partial \Theta_l}{\partial t} = - \frac{\partial \langle w\Theta_l \rangle}{\partial z} - \frac{\Theta L \partial G}{T c_p \partial z}, \tag{3}
\]

\[
\frac{\partial Q_w}{\partial t} = - \frac{\partial \langle wq_w \rangle}{\partial z} + \frac{\partial G}{\partial z}, \tag{4}
\]

where \(U\) and \(V\) are the horizontal wind components, \(\langle uw \rangle\) and \(\langle vw \rangle\) the (kinematic) shear stress components, \(f\) the Coriolis parameter, \(U_g\) and \(V_g\) the geostrophic wind components, \(\Theta_l[\equiv \Theta - (\Theta/T)(L_v/c_p)Q_l]\) the liquid-water potential temperature, \(\Theta\) the potential temperature, \(T\) the absolute temperature, \(c_p\) the specific heat of dry air at constant pressure, \(L_v\) is the latent heat of condensation/freezing or evaporation/sublimation for water vapour, \(G\) the gravitational settling flux (positive downward) of fog droplets, \(Q_w(\equiv Q_v + Q_l)\) the total-water content, \(Q_v\) the specific humidity, \(Q_l\) the liquid-water content, \(\langle w\Theta_l \rangle\) the (kinematic) liquid-water potential temperature flux (positive upwards) and \(\langle wq_w \rangle\) the total-water content flux (\(\langle wq_v \rangle\) and \(\langle wq_l \rangle\) the (kinematic) moisture and liquid-water fluxes respectively). Note the effects of the radiation is neglected here.
\( \Theta_l, \Theta_w \) and \( Q_w \)

\( \Theta_l \equiv \Theta - (\Theta/T)(L_v/c_p)Q_l \) the liquid-water potential temperature

potential temperature of dry air with same density as moist air + water droplets - needed for stability considerations and TKE equation.

\( \Theta_v \equiv \Theta(1 + 0.61Q_v - Q_l) \)

\( Q_w \equiv Q_v + Q_l \) the total-water content, \( Q_v \) the specific humidity, \( Q_l \) the liquid-water content.
TKE and closure

\[
\langle uw \rangle = -K_m \frac{\partial U}{\partial z}, \quad \langle vw \rangle = -K_m \frac{\partial V}{\partial z}, \quad (5)
\]

\[
\langle w\theta_l \rangle = -K_h \frac{\partial \Theta_l}{\partial z}, \quad \langle wq_w \rangle = -K_q \frac{\partial Q_w}{\partial z}, \quad (6)
\]

\[
K_m = (\alpha E)^{1/2} \ell_m, \quad K_h = K_q = (\alpha E)^{1/2} \ell_m / Pr, \quad (7)
\]

\[
\frac{\partial E}{\partial t} = -\langle uw \rangle \frac{\partial U}{\partial z} - \langle vw \rangle \frac{\partial V}{\partial z} + \beta g \langle w\theta_v \rangle - \epsilon + \frac{\partial}{\partial z} \left( K_m \frac{\partial E}{\partial z} \right), \quad (8)
\]

for neutral and unstable conditions,

\[
\frac{1}{\ell_m} = \frac{1}{\ell_d} = \frac{\phi_m}{\kappa (z + z_0)} + \frac{1}{\ell_0}, \quad (9)
\]

for stably stratified flow,

\[
\frac{1}{\ell_m} = \frac{1}{\kappa (z + z_0)} + \frac{1}{\ell_0} + \frac{\beta_c}{\kappa L_0}, \quad (10)
\]

\[
\frac{1}{\ell_d} = \frac{1}{\kappa (z + z_0)} + \frac{1}{\ell_0} + \frac{\beta_c - 1}{\kappa L_0}, \quad (11)
\]
More closure details

\[
\phi_m = \begin{cases} 
1 + \beta_c z/L_o, & \text{for } z/L_o \geq 0; \\
(1 - \gamma_1 z/L_o)^{-1/4}, & \text{for } z/L_o < 0,
\end{cases}
\]  
(12)

where constants \(\beta_c = 4.7\), \(\gamma_1 = 15\) and \(L_o\) is the Obukhov length, which is defined as

\[
L_o = -\frac{u_*^3}{\kappa \beta g \langle w\theta_v \rangle}.
\]  
(13)

\[
Q_l = \begin{cases} 
a (Q_w - Q_{sl}), & \text{for } Q_w > Q_{sl}; \\
0, & \text{for } Q_w \leq Q_{sl},
\end{cases}
\]  
(14)

\[
a = \frac{1}{1 + \delta Q_{sl} L_v/c_p}
\]  
and

\[
\delta Q_{sl} = \left. \frac{\partial Q_s}{\partial T} \right|_{T = T_l} = 0.622 \frac{L_v Q_{sl}}{R_d T_l^2},
\]  
(15)

\(Q_s\) is the saturation specific humidity, \(Q_{sl} \equiv Q_s(T_l)\) and \(T_l = \Theta_l T/\Theta\).
1-D modelling strategy

- Start from a steady state neutral PBL model - $U_g$, $f$, $z_0$ specified - steady state, horizontally homogeneous. Run for several inertial cycles.

- Change temperature profile ($dT/dz = -4K/km$) and add water vapour (90% relative humidity) and run for 1 inertial cycle. Surface 100% RH.

- Start surface cooling (5 K/hr - rather high but as substitute for advection it is OK)

- Basic Case ($U_g = 10 \text{ ms}^{-1}$, $V_T = 0$ or $10^6(Q_l/N_0)^{2/3} \text{ ms}^{-1}$)

- Vary $U_g$, 5 ms$^{-1}$, 20 ms$^{-1}$.
Basic Case, $V_T = 0$, cooling 5K/hr
Basic Case, $V_T = 0$, cooling 5K/hr

![Diagram showing height vs. $Q_w$ and $Q_l$.]

- **Height (m)**
  - Initial (I)
  - Initial (II)
  - 1 hr
  - 2 hr
  - 3 hr

- **$Q_w$ (g kg$^{-1}$)**

- **$Q_l$ (g kg$^{-1}$)**
Basic Case, $V_T = 0$, cooling 5K/hr

- Initial (I)
- Initial (II)
- 1 hr
- 2 hr
- 3 hr
Basic Case, $V_T = 0$, cooling 5K/hr
Basic Case, $V_T$ added, cooling 5K/hr

- Initial (II)
- 0.5 hr
- 1 hr
- 1.5 hr
- 2 hr
- 2.5 hr
- 3 hr

Height (m)

$Q_w$ (g kg$^{-1}$)

$Q_i$ (g kg$^{-1}$)
Ug = 20 ms$^{-1}$, $V_T$ added, cooling 5 K/hr
$U_g = 5\text{ms}^{-1}, V_T$ added, cooling 5K/hr

![Graph showing height vs. $Q_w$ and $Q_t$](image-url)
Next Steps

- Add radiation terms
- Resolve issues with settling and lower boundary condition for $Q_l$
- Get data on initial profiles with/without fog.
- Get the advective case running
- Determine combinations of conditions (upstream profiles, wind speeds, cloud cover, rate of change of surface temperature, etc. etc.) under which fog forms, and fog dissipation conditions.
Still struggling to clear the fog but making progress in understanding the conditions that allow fog to form.