Peter Taylor 16 Feb 2021 Atmospheric Research Group

Marine fog , constant flux layers and WRF

A group of us, including Zheqi, Li, Soudeh, Wensong, George, Yongsheng and I have been looking for ways to improve WRF forecasts of fog and quantitative visibility predictions offshore from Atlantic Canada. Getting into the details of how fog/cloud droplet water content is treated within the surface boundary layer led us to look at deposition of fog droplets towards underlying ground/water surfaces. This can be a mix of gravitational settling and turbulence causing droplets to impact the surface. WRF can include turbulent deposition for vegetation but, until we added it, was not representing turbulent deposition velocities for water surfaces. For neutral, boundary-layer log profiles one can find a neat profile for a **constant flux layer with gravitational settling, CFLGS.**

Surface deposition of marine fog and its treatment in the WRF model: Peter A. Taylor • Zheqi Chen • Li Cheng • Soudeh Afsharian • Wensong Weng • George A. Isaac • Terry W. Bullock • Yongsheng Chen



Observations show more than 200 (out of 720) hours of fog (visibility < 1 km) on Sable Island in the months of June and July.





Characterizing and Predicting Marine Fog Offshore Newfoundland and Labrador

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FIG. 12. Liquid water content vs droplet diameter for LWC ranges of 0.005– 0.01, 0.01–0.05, 0.05–0.1, and greater than 0.1 g m23 for (c) 2018.

Visibility, V_k (km), contrast threshold, $\epsilon = 0.05$

$$\begin{split} V_k &= \frac{-\ln(\varepsilon)}{\beta}, \\ \beta &= \frac{3 \text{LWC}}{\rho_w D_{\text{eff}}}, \\ \text{LWC} &= \rho_w N \frac{\pi}{6} D_{\text{eff}}^3, \\ V_k &= \frac{1.24 \rho_w^{2/3}}{\text{LWC}^{2/3} N^{1/3}}, \end{split}$$

For a given LWC, $V_k \sim D_{eff}$ But N and size distribution should also come into play. Gravitational Settling Velocity

Fog droplets have a range of sizes but most fall in the diameter range 0-50 µm, often with bimodal distributions and peaks around 6 and 25 µm (see for example Isaac et al, 2020). Applying Stokes law, $w_s = gd^2(\rho_w - \rho_a)/(18\mu)$, where $v = \mu/\rho_a = 15.06 \times 10^{-6} m^2 s^{-1}$ at 20°C and standard pressure, is the kinematic viscosity of air with saturated density, ρ_a (1.178 kg m-3), water droplet density, ρ_w and acceleration due to gravity, g, for these peak sizes gives w_s values of 0.0011 and 0.0192 m s⁻¹. These terminal velocities are clearly small compared to wind speed but for the larger diameter droplets, where the bulk of the liquid water content, LWC (= ρ_a Qc), is often measured, the terminal velocity corresponds to 69 m per hour and will represent a considerable removal rate in fog which may last several hours or days.

What happens at the water surface? Gravitational settling PLUS turbulent impact? What do various models assume, especially WRF.

Bounce ($\rightarrow \partial Qc/\partial n=0$), Collide and Coalesce ($\rightarrow Qc=0$)

The Atmospheric Boundary Layer

- Surface layer, constant flux layer, momentum and heat. Turbulence!
- The planetary boundary layer, Coriolis effects. Inversion capped. Generally time dependent.
- Stratification effects, Monin-Obukhov Similarity
- Roughness length and roughness elements
- Horizontal homogeneity, steady state or diurnal cycle.
- Flat terrain and topographic effects
- Radiative flux divergence, clouds and fog.
- We generally live and work in it many important applications, including surface fluxes and air quality.

The steady state atmospheric surface layer over homogeneous flat terrain.

Monin-Obukhov Similarity Theory – MOST, 1946, 1954

Constant flux layers - momentum (ρu^{*2}) and heat (H/(ρc_p) or buoyancy fluxes <w'p'>, or fog liquid water mixing ratio (Qc) or density (LWC).

Relationship between gradients and fluxes,

 $\partial U/\partial z = (u^*/kz) \Phi_M(\zeta)$ or $(kz/u^*) \partial U/\partial z = \Phi_M(\zeta)$ - both dimensionless.

Buckingham Pi theorem: $\pi_1 = (kz/u^*)\partial U/\partial z$: $\pi_2 = z/L = \zeta$, where $L = -u^{*3}/[k(g/\theta) < w'\theta' >]$ is the Obukhov length $\pi_1 = F(\pi_2)$.

We argue that g affects buoyancy only via g/ θ so can use that combination, then, for wind speed, consider 5 dimensional variables: $\partial U/\partial z$, u^{*}, z, g/ θ , H/(ρc_p). 3 Dimensions L,t,K so 2 π s as above. Furthermore any dimensionless turbulence property will be a function of ζ alone, e.g, $\langle w'^2 \rangle / u^{*2}$ or $\partial \Theta/\partial z = (\theta^*/kz) \Phi_H(\zeta)$.

Why is this important? Provides universal relationships which can be determined experimentally.

With neutral stratification ($\zeta = z/L = 0$, $\Phi_M(0) = 1$) we get log velocity, and other profiles. $\partial U/\partial z = (u^*/kz) \Phi_M(\zeta) = (u^*/kz)$ with u* constant, U = 0 at z = z_{0m} . Note $z_{0m} << z$. $U(z) = (u^*/k) \ln(z/z_{0m})$, or, with U = 0 at z = 0, $U = (u^*/k) \ln((z+z_{0m})/z_{0m})$



Constant Flux Layers with Gravitational Settling: deposition to an underlying surface and links to fog.

Consider an idealized situation where the lowest layers of a horizontally homogeneous boundary-layer fog situation are at 100% relative humidity, are in a steady state and could be considered as having a constant downward flux of uniform size cloud droplets and associated liquid water mixing ratio. One could then model the constant downward flux of fog water, F_{Oc} , with an equation

$$w_s Qc + ku^*(z + z_{0c}) dQc/dz = F_{Qc} = u^* q_c^*,$$
 (3)

where w_s represents the gravitational settling velocity. The eddy diffusivity K_{qc} is assumed to be $K_{qc} = ku^*(z + z_{0c}),$ (4)

where z_{0c} is a roughness length for fog droplets with the assumption that $Qc = Qc_{surf}$ at z = 0. Over a water surface we assume $Qc_{surf} = 0$. Initially we can assume a **single drop size, with a single** w_s **and single** z_{0c} but, provided that individual drops retain their size and integrity as they pass through the constant flux layer one can apply these ideas to multiple size bins and combine the profiles of each to get Qc(z) totals.

Assuming constant values for z_{0c} , u^* and w_s one can then solve the first order ODE, Eq (3), by integrating factor techniques, multiplying (3) by $(z+z_{0c})^{S-1}/(ku^*)$ where $S = w_s/(ku^*)$, to give,

$$(d/dz)[(z+z_{0s})^{S}Qc] = (q^{*}/k)(z+z_{0c})^{S-1}$$
(5)

and, with Qc(0) = 0

$$Qc(z) = (q_c */(kS)) [1 - ((z + z_{0c})/z_{0c})^{-S}]$$
(6)

or, in terms of $\xi = \ln ((z+z_{0c})/z_{0c})$, note ξ because $\zeta = z/L$.

$$Qc(\xi) = (q_c */(kS)) [1 - e^{-S\xi}].$$
 (7)

These can be referred to as Constant Flux Layer with Gravitational Settling, CFLGS, profiles. In the limit as w_s and $S \to 0$, as $\zeta \to 0$, Eq (7) would give $Qc(\xi) = (q_c */k) \xi$, a standard log profile.

The key parameter in our constant flux with gravitational settling model is $S = w_s/ku^*$. In moderate winds over the ocean one might expect u^* values in the 0.2-0.5 m s⁻¹ range, while in radiation fog in light winds over land it could be lower, maybe 0.1 ms⁻¹. Recall that typical w_s values are < 0.02 ms⁻¹. The parameter, S will thus generally be in the range 0 – 0.5.

Over water with moderate winds (say 10 ms⁻¹) we might assume a near constant flux up to 50m and with $z_{0m} = 0.0001$ m and U(50m) = 10 ms⁻¹ we have u* = 0.305 ms⁻¹ and for our small and large droplets (w_s = 0.0011 and 0.0192 ms⁻¹) **S** = 0.009 and 0.157.



Fig. 1 *Qc* profiles, scaled by 50 m value, from surface to z = 50 m in constant flux layers with gravitational settling and surface roughness length for water droplet removal, $z_{0c} = 0.1$ m. Linear (a) and logarithmic (b) height scales. (zeta) $\xi = ln((z+z_{0c})/z_{0c})$

Recall typical marine fog values S = 0.009 and 0.157.

What is z_{0c} ?





Fig. 2 Variation of the Turbulent Transfer fraction of the total Qc flux and its variation with z and S. Note that these z values are based on (left) $z_{0c} = 0.001$ m and (right) $z_{0c} = 0.1$ m *TTratio* = *Turbulent/Total flux* = $e^{-S\xi}$

Bottom line – Some simple math gives an interesting new result – at least I think it is new. But in more complex situations what is the impact of turbulent Qc flux to the surface?







Fig. 5. Sample 3-D WRF output at a fixed location over the Grand Banks, with different z_{0c} values in Qc turbulent deposition, a) with and b) without gravitational settling. Start time was 7/1 12Z, 2018 and results are for 7/1 18Z. Results are with MYNN boundary layer and Thompson microphysics.



If this removal process needs to be included in modelling and forecasting fog occurrence we need to know more about it. Fog is intermittent so setting up 50-m or higher measurement masts in fogprone locations will be a good start. Tethered kite or balloon and UAV profile campaigns would be useful to extend the height range. Multiple measurement levels are needed, measuring droplet size distributions, Qc or LWC values and ideally Qc fluxes, along with wind, turbulence, temperature and humidity profiles plus surface fluxes of momentum, heat and water vapour. Visibility at multiple levels, 4 component radiation and air, aerosol and fog chemistry measurements could play an important role.

Over forests Katata et al have done this sort of thing for deposition velocities, but there seems to have been nothing similar over water. Sable Island would be an ideal place for such a study!



From the modelling perspective we need values for z_{0c} , which will depend on surface type and probably on droplet diameter and on wind speed or friction velocity. Assuming that the lower layers, say 10-30 m of a deep fog layer, are in a steady, constant flux layer situation then the CFLGS profiles developed above could provide a framework for analysis of observations. They could be combined for multiple size bins and potentially extended to include non-neutral stratification via Monin-Obukhov similarity theory.

MOST for stably stratified boundary layers

 $\Phi_{M}(\zeta) = 1 + \beta (z + z_{0m})/L : U = (u^{*}/k) (ln ((z + z_{0m})/z_{0m}) + \beta (z + z_{0m})/L, Generally \beta = 5 and K_{m} = k(z + z_{0m})/\Phi_{M}(z/L)$

Suppose we extend this to $K_{Qc} = k(z+z_{0c})/\Phi_{Qc}(z/L)$. Can we solve,

 $w_sQc + [ku^*(z + z_{0c})/\Phi_{Qc}(z/L)] dQc/dz = F_{Qc} = u^*q_c^*, \text{ or}$

 $dQc/dz + S(1+\theta (z+z_{0c})/L)/(z+z_{0c})Qc = (q_c*/k)(1+\theta(z+z_{0c})/L)/(z+z_{0c}); S=w_s/(ku*)$

The Integrating Factor is $\exp(\int S(1/(z+z_{0c})+\beta/L)dz = (z+z_{0c})^{s} \exp(\beta z/L)$

So that $d[(z+z_{0c})^{s} exp(\theta z/L)Qc]/dz = (q_{c}*/k)(1+\theta(z+z_{0c})/L)(z+z_{0c})^{s-1} exp(\theta z/L)$

And I need integrals of the form $\int x^{\alpha} e^{x} dx$ for $-1 < \alpha < 1$ and $x > z_{0c}$. - working on it now – may be possible with exponential integrals