BOUNDARY LAYER FLOW OVER HILLS

Peter Taylor, ESSE, York University

P.S. Jackson, J.C.R. Hunt, Turbulent wind flow over a low hill, Quart. J. Roy. Met. Soc. 101, 929-955 (1975).

> “There is clearly a need for a simple analytical theory which is able to predict the general features of the effect of a small hump on a turbulent boundary layer and to demonstrate the influence of changes in the physical parameters determining the flow. ‘

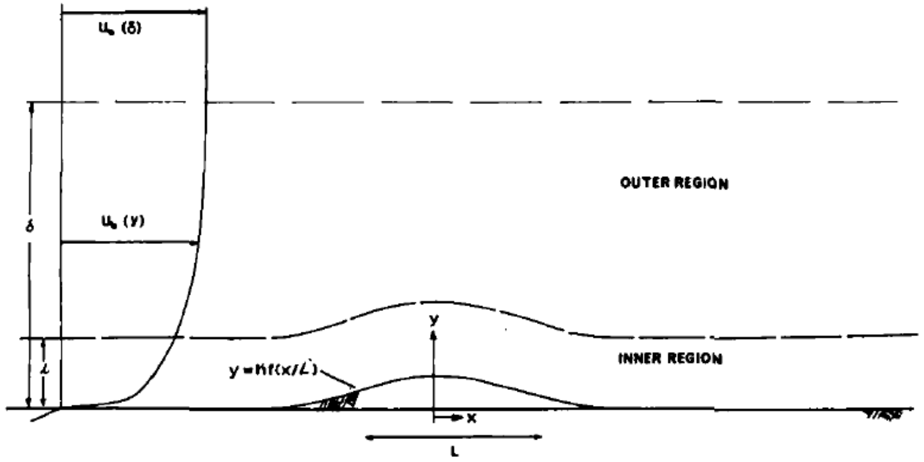


Figure 1. Flow regimes for turbulent flow over a low hill.

Jackson and Hunt [1] divide the flow over low hills into inner and outer layers. (There are also triple decks …… Belcher….) In the outer layer, perturbations to the shear stress associated with the flow over the hill are assumed to be of no dynamical significance and the flow can be treated as essentially inviscid. The inner layer, of depth l_I, is defined by the height at which the perturbation stress gradient, induced by flow over the hill, is of the same magnitude as the non-linear advection term ($U\partial U/\partial x$). The two terms combined will in turn approximately balance the pressure gradient if, for the moment, we think in streamline coordinates (s,n) and ignore the vertical advection term.

This leads to $(l_I/L)\ln(l_I/z_0) = 2\kappa^2$ although Jensen argues for $(l_I/L)[\ln(l_I/z_0)]^2 = 2\kappa^2$ which makes it shallower. Depths will depend on L/z_0, where z_0 is roughness length.

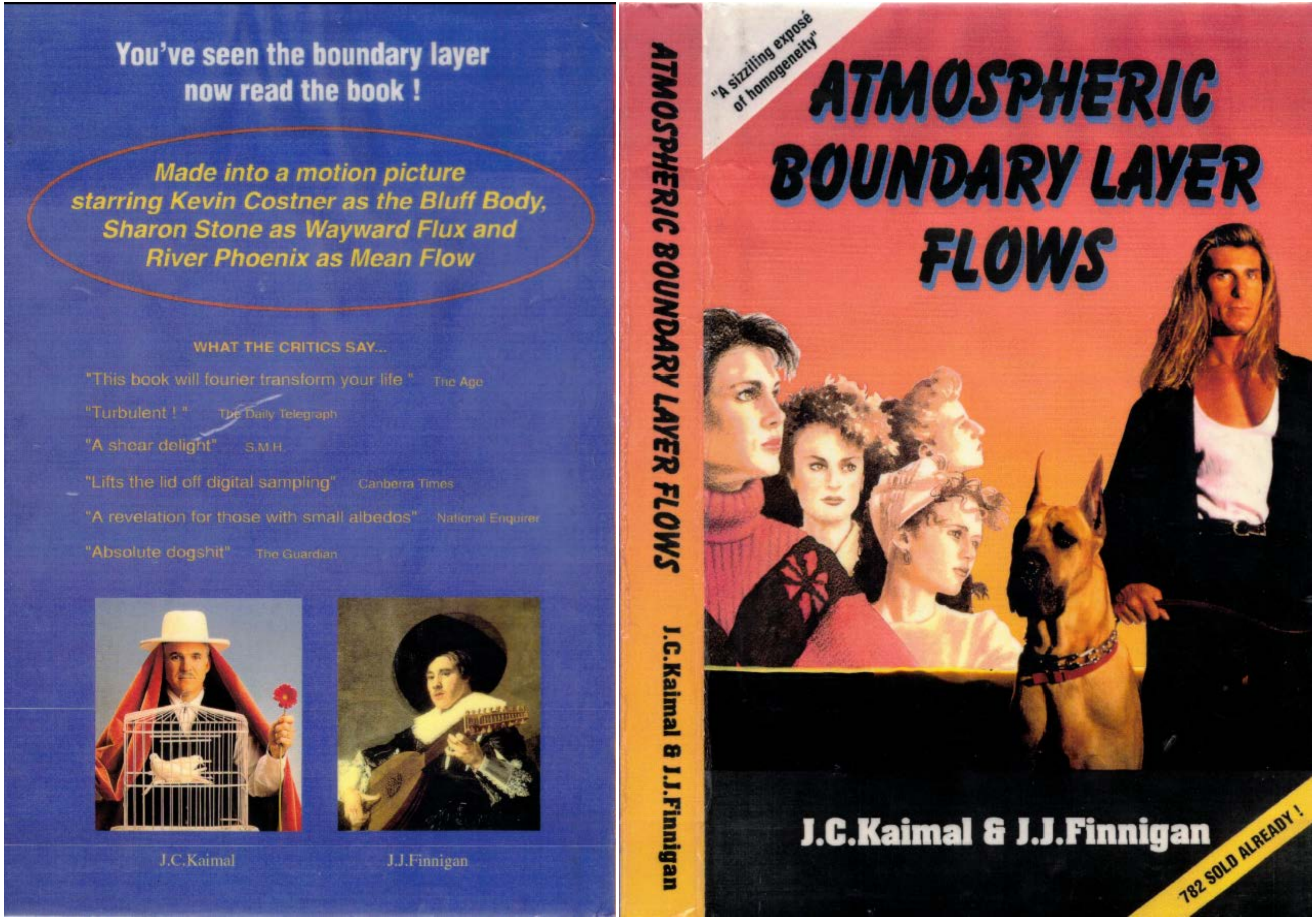
Note that for Askervein we assume $z_0 = 0.03$ m. L was a function of wind direction, but with a minimum of 215 m. With these values l_I is 11.6 m while l_{NOJ} is 3.2 m — both small compared to the dimensions of the hill.

A DuckDuckGo search of “Boundary-Layer Flow over Hills” yielded many links but **the top 2** were

https://academic.oup.com › book › 40882 › chapter 5› 348956444 ---- Kaimal and Finnigan’s book Flow Over Hills | Atmospheric Boundary Layer Flows: Their Structure and Measurement | Oxford Academic

and

https://link.springer.com › article › 10.1007 › s10546-020-00564-3 ------- Finnigan et al’s recent (2020) review. Boundary-Layer Flow Over Complex Topography | SpringerLink 2477 Accesses, 41 Citations



You’ve seen the boundary layer
now read the book !

Made into a motion picture
starring Kevin Costner as the Bluff Body,
Sharon Stone as Wayward Flux and
River Phoenix as Mean Flow

WHAT THE CRITICS SAY...

"This book will fourier transform your life " The Age

"Turbulent ! " The Daily Telegraph

"A shear delight" S.M.H.

"Lifts the lid off digital sampling" Canberra Times

"A revelation for those with small albedos" National Enquirer

"Absolute dogshit" The Guardian

J.C.Kaimal

J.J.Finnigan

ATMOSPHERIC BOUNDARY LAYER FLOWS

J.C.Kaimal & J.J.Finnigan

"A sizzling exposé of homogeneity"

ATMOSPHERIC
BOUNDARY LAYER
FLOWS

J.C.Kaimal & J.J.Finnigan

782 SOLD ALREADY !

In 3rd place comes another, rather older (1987) review,
https://link.springer.com/article/10.1007/BF00121870
Boundary-layer flow over low hills: P. A. Taylor, P. J. Mason & E. F. Bradley in
Boundary-Layer Meteorology volume 39, pages 107–132 (1987), 840 Accesses, 158 Citations

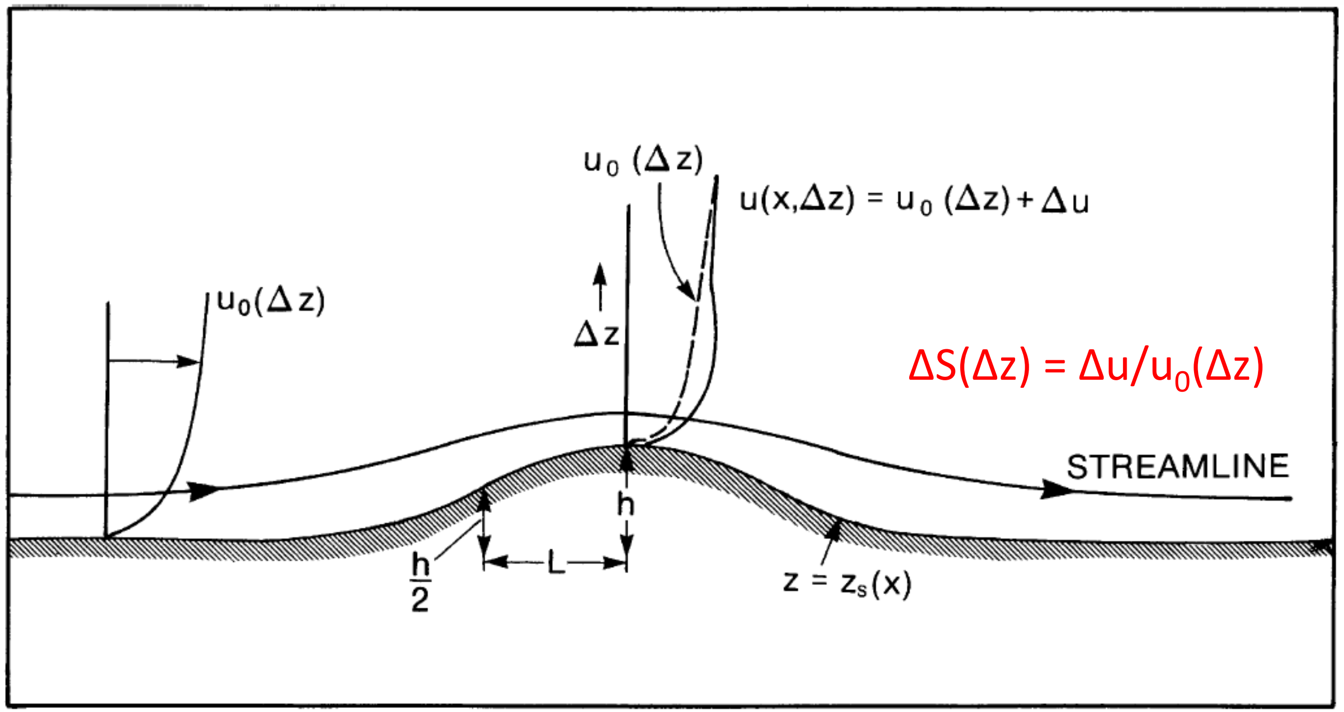
A wind energy thought:

$P \sim A\rho U^3$

Sometimes, #4 in the list is my review from 1998,
https://www.sciencedirect.com › science › article › pii › S0167610598000051
Journal of Wind Engineering and Industrial Aerodynamics, Volumes 74–76, 1 April 1998, Pages 25-47

My 1998 review opens with, “About twenty five years ago several groups started work on the theory and modelling of boundary-layer flow over hills. Hunt and Jackson, both then at Cambridge, introduced [1] the important concept of the "inner layer", of depth l I, and provided definitions of the fractional speed-up ratio, ΔS, and the characteristic length scale of a hill for a given wind direction, *L*, the upstream distance to the point where the elevation was half the maximum”

2023 = 1998 = 25, so 50 years ago ….



$\Delta S(\Delta z) = \Delta u / u_0(\Delta z)$

It can be argued that before about 1960 almost all boundary layer studies sought out flat, homogeneous terrain and assumed long enough fetches upwind of any measurement sites that, in the lowest ~50m a “constant flux layer” approach was justified. In this layer Coriolis forces were neglected. The same constant flux layer approach is a part of Monin-Obukhov Similarity Theory (MOST).

A few quotes from Finnigan et al (2020).
“It is over 70 years since the similarity theory of Monin and Obukhov (MOST) was developed in the USSR and over 50 years since experiments on the sweeping plains of South-Eastern Australia and the mid-West of the USA validated it for the first time”.
“However, at the most fundamental level, MOST applies only to flat homogeneous terrain and most of the Earth’s surface is not flat but topographically complex on scales from hillocks to mountains so it is no surprise that the study of airflow over hills and valleys has a history as long as MOST.”

My introduction to heterogeneity came through Ted Munn’s book (1966, Descriptive Micrometeorology) where 5 pages in Section 12.3 deal with “The effect of a discrete change in roughness”. Munn also pointed out (Section 11.1) that “A homogeneous infinite plane is the exception rather than the rule in nature. The countryside is often hilly ….” but he then focusses on wind flow around obstacles. My 1967 PhD “On turbulent wall flows above a change in surface roughness” dealt with heterogeneity but not hills. Interest in those started later at Southampton working with Peter Gent on flow over both 2-D hills and water waves (1974-1978). Moving to work at Environment Canada, John Walmsley got me to read the Jackson and Hunt (1975, QJRMS 101,929-955) paper, and its 3D extension by Mason and Sykes (QJRMS 105:383–395). We “built” the MS3DJH model.

John Finnigan’s previous review of “Airflow over complex terrain” was in a the proceedings of a 1987 symposium in Canberra, that I was lucky enough to attend. Published in Steffen and Denmead, 1988, Flow and Transport in the Natural Environment, Springer-Verlag, may be relatively hard to find.

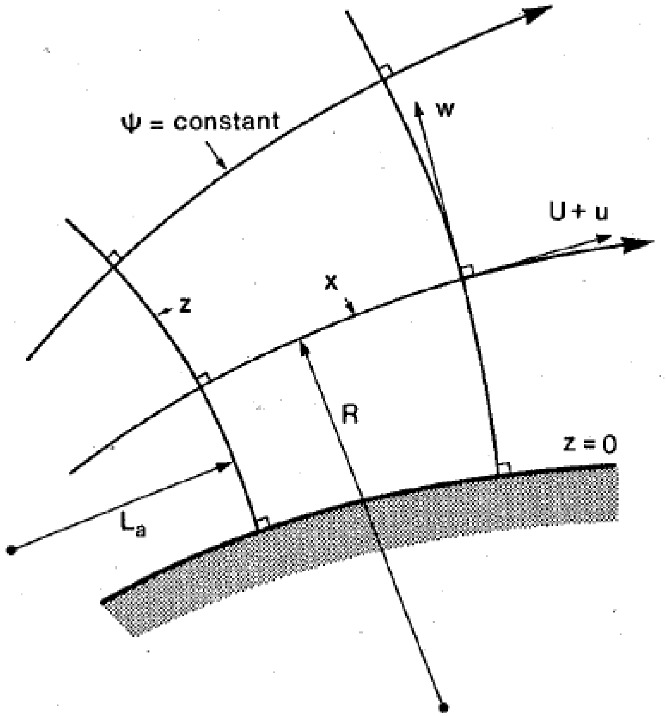


Fig. 1. The 2-D streamline coordinate system. The curves Ψ = constant, Ψ being the stream function, are the x coordinate lines. Their orthogonal trajectories are the z lines. The streamline at z = 0 is the surface. R and L_a are the local radii of curvature of the x and z lines.

JJF’s 1987/8 review included discussion of wind tunnel and field measurements (Black Mountain, Coopers Ridge, Blasheval and Askervein but also highlighted the potential role of streamline curvature and promoted the use of streamline coordinates, following on from his 1983 JFM paper (130, 241-258) on that topic.

Streamline curvature is also Chapter 5 of K&F, with extension to 3D flows. Key equations are

$$U\partial_x U = -\frac{1}{\rho}\partial_x P - \partial_x\overline{u^2} + \frac{\overline{u^2}-\overline{w^2}}{L_a} - \partial_z\overline{uw} + \frac{2\overline{uw}}{R} - \nu\partial_z\Omega$$

$$\frac{U^2}{R} = -\frac{1}{\rho}\partial_z P - \partial_x\overline{uw} + \frac{2\overline{uw}}{L_a} - \partial_z\overline{w^2} + \frac{\overline{u^2}-\overline{w^2}}{R} + \nu\partial_x\Omega \ ,$$

Also need to consider role in turbulence and closure schemes…..

From Finnigan (1988), 2D Reynolds stresses, u=in curvilinear coordinates, are

$$\text{Production } \overline{u^2} = -2\left[\overline{u^2}\,\frac{U}{L_a} - \overline{uw}\,\frac{U}{R} + \overline{uw}\,\partial_z U\right] ,$$

$$\text{Production } \overline{w^2} = -2\left[2\overline{uw}\,\frac{U}{R} - \overline{w^2}\,\frac{U}{L_a}\right] ,$$

$$\text{Production } \overline{uw} = -\left[(2\overline{u^2} - \overline{w^2}) + \overline{w^2}\,\partial_z U\right] .$$

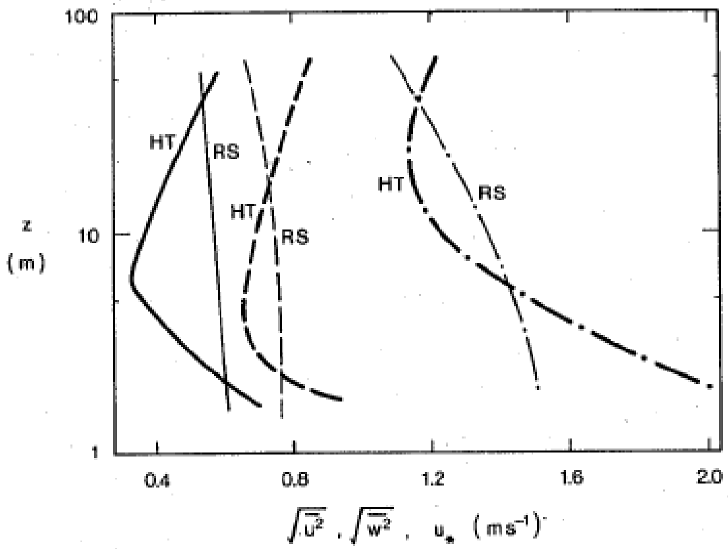


Fig. 16. Vertical profiles of $(\overline{u^2})^{\frac{1}{2}}$ (—·—); $(\overline{w^2})^{\frac{1}{2}}$ (— —) and u_* (= $(-\overline{uw})^{\frac{1}{2}}$) (——) from two positions on Askervein Hill with the wind perpendicular to the long axis of the hill (235°). HT denotes hill top and RS the upwind reference station. (Mickle et al. 1988).

And on Askervein, almost a 2D ridge, the Niels-Otto Jensen and the Riso group had a 50m tower on the hilltop and sonic anemometers measuring turbulence.

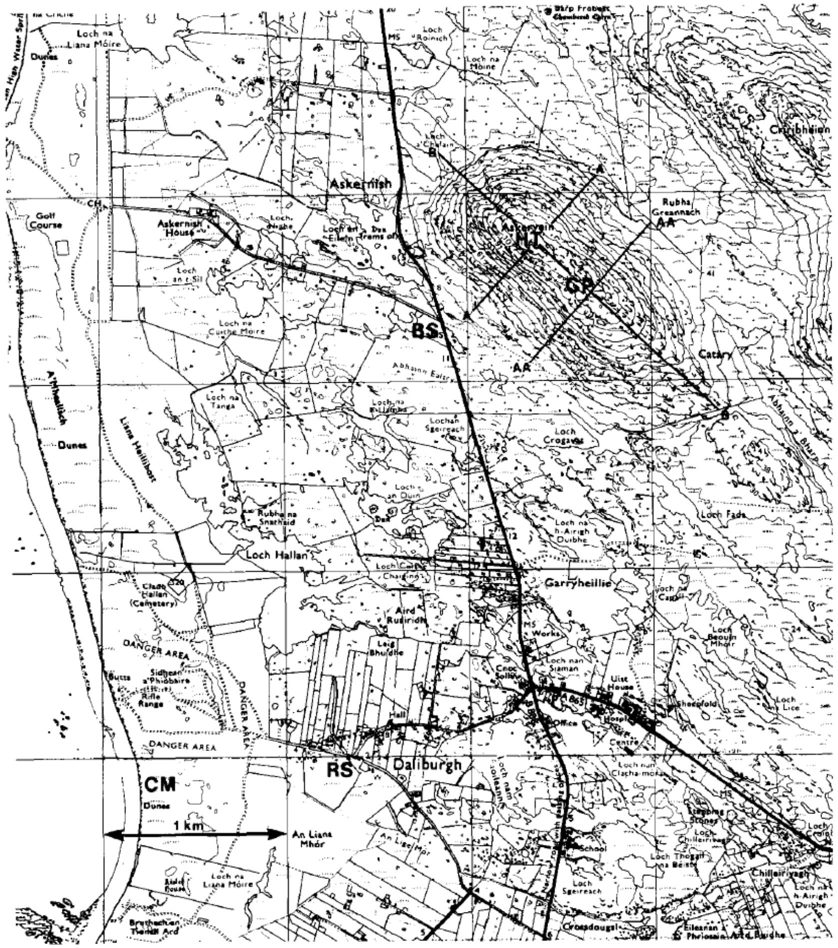
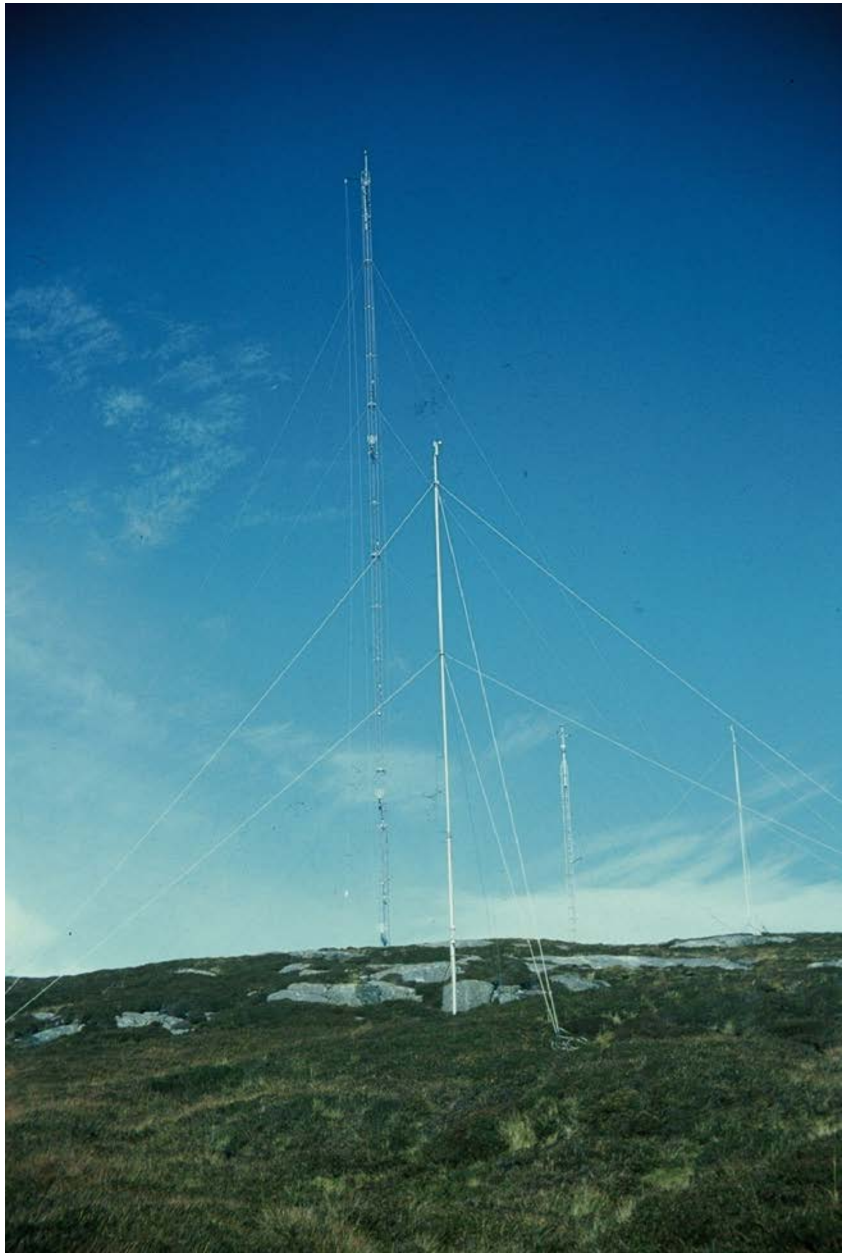
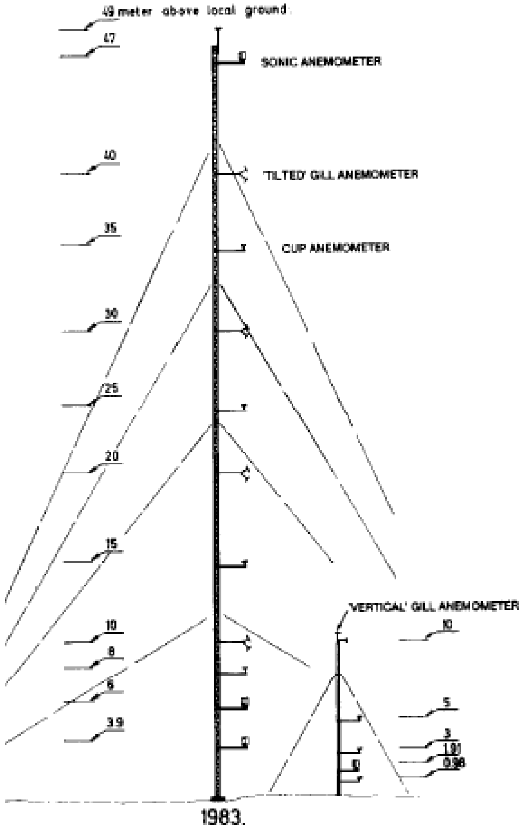


Fig. 1. Contour map of Askervein Hill and surrounding terrain. Contour intervals are 7 or 8 m (25 ft.). HT–Hilltop, CP–Centre Point, BS–Base Station, RS–Reference Site, CM–Coastal Machair Site (Machair–a flat or low lying plain or field–Scottish Gaelic)





Pictures and reports at https://www.yorku.ca/pat/research/Askervein/

Many participants, IEA encouragement. Unsal Hassan, ERA …

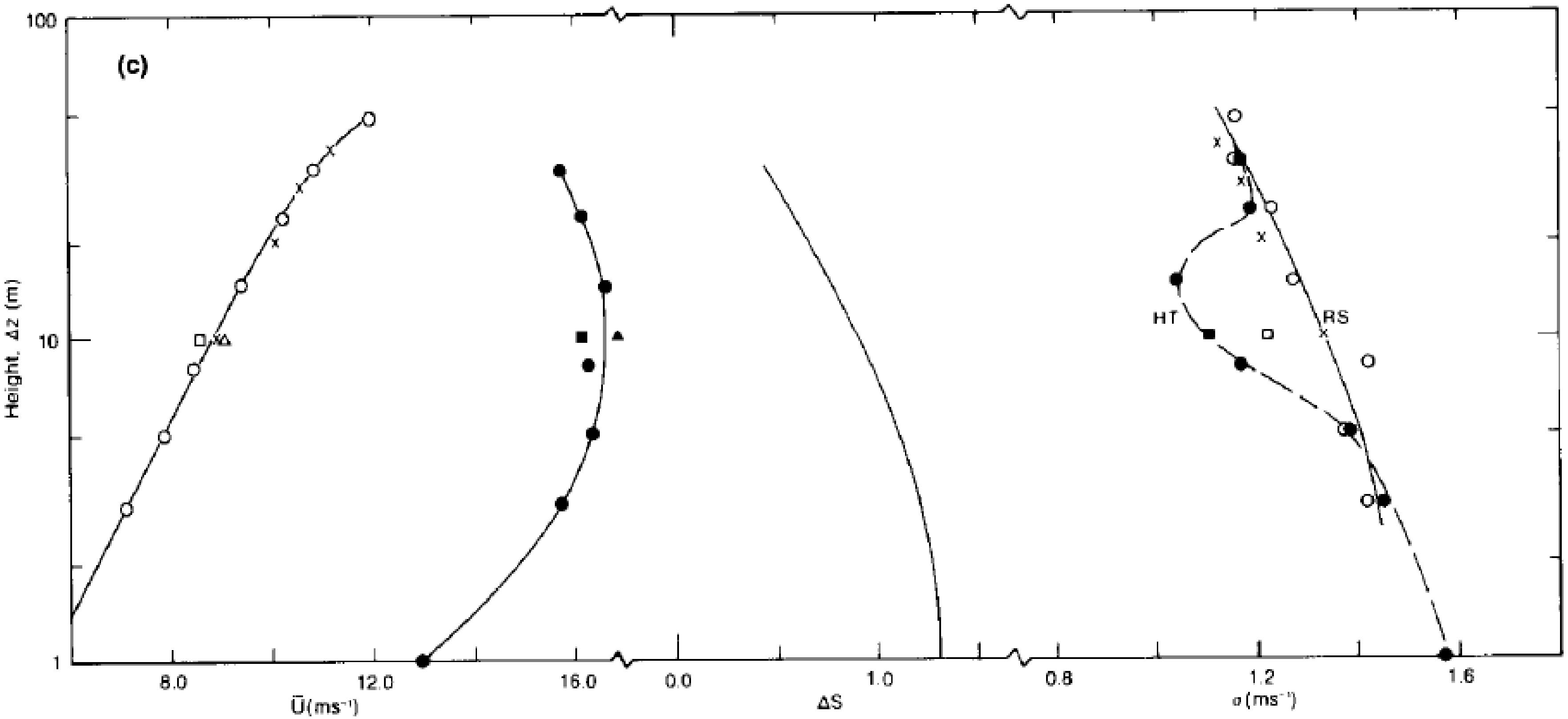


Fig. 6. Mean wind speed, ΔS and σ_h profiles at RS and HT during selected TU runs, Askervein '83. (a) Run TU30-B, $\varphi \approx 130°$. (b) Run TU01-B, $\varphi \approx 180°$. (c) Run TU03-B, $\varphi \approx 210°$. (d) Run TU07-A, $\varphi \approx 240°$. (e) Run TU05-B, $\varphi \approx 305°$.

Cup anemometers	○ RS	● HT
Tilted Gill UVW	× RS	
Vertical Gill UVW (10 m)	□ RS	■ HT
MF Posts (10 m)	△ RS	▲ HT

Mean profiles and ΔS – fractional speed-up ratio wre a central goal. But with cup anemometers, and other instruments we measured wind speed variances. So above hilltop,
U increases but σ_h decreases.

Rapid Distortion Theory (RDT) at work, but the JJF 1988 review (p214) points out misinterpretations and failures to account for streamline curvature effects. Note σ_w also decreases.

Spatial variations – just mean winds, relative to winds at RS. Averages of several half hour runs, and standard deviations.

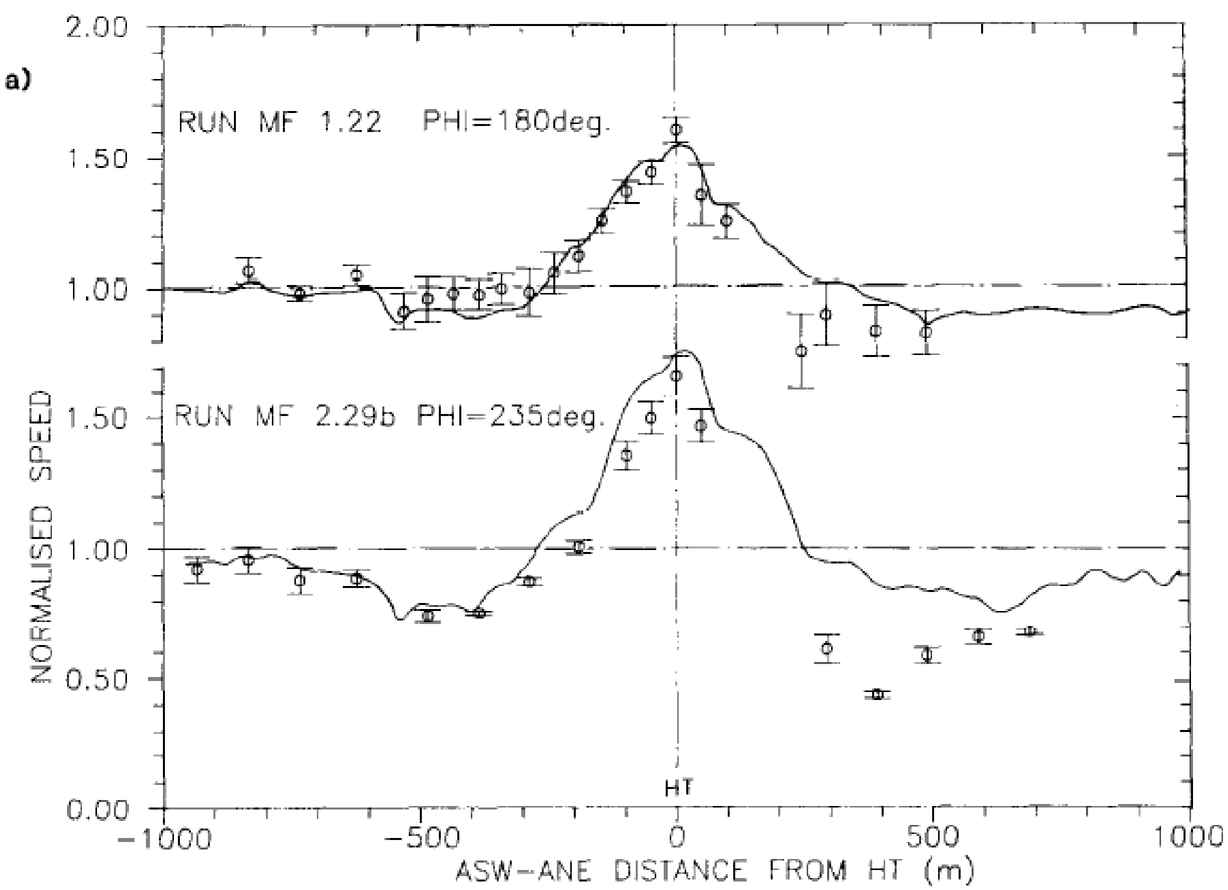


Fig. 6a.

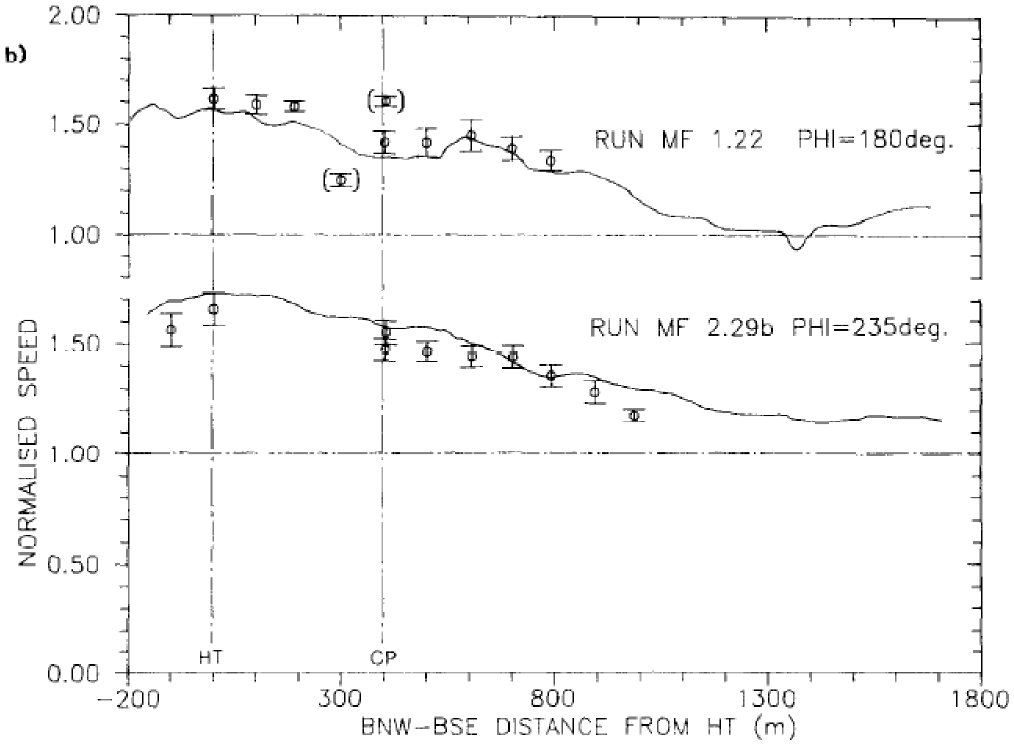


Fig. 6b.

Fig. 6. Sample plots of normalized wind speed at $\Delta z = 10$ m along A and B lines, Askervein '82. (a) A line; (b) B line. Ɪ mean of 30 min values, error bars denote one standard deviation; ( ) questionable data; —— MS3DJH/3 model results.

Lines are MS3DJH model results – less satisfactory in lee of hill. Surface layer model.

Weng, W. and Taylor, P. A., 2011: A Non-Linear Mixed Spectral Finite-Difference 3-D model for planetary boundary-layer flow over complex terrain, Adv. Sci. Res., 6, 75-7

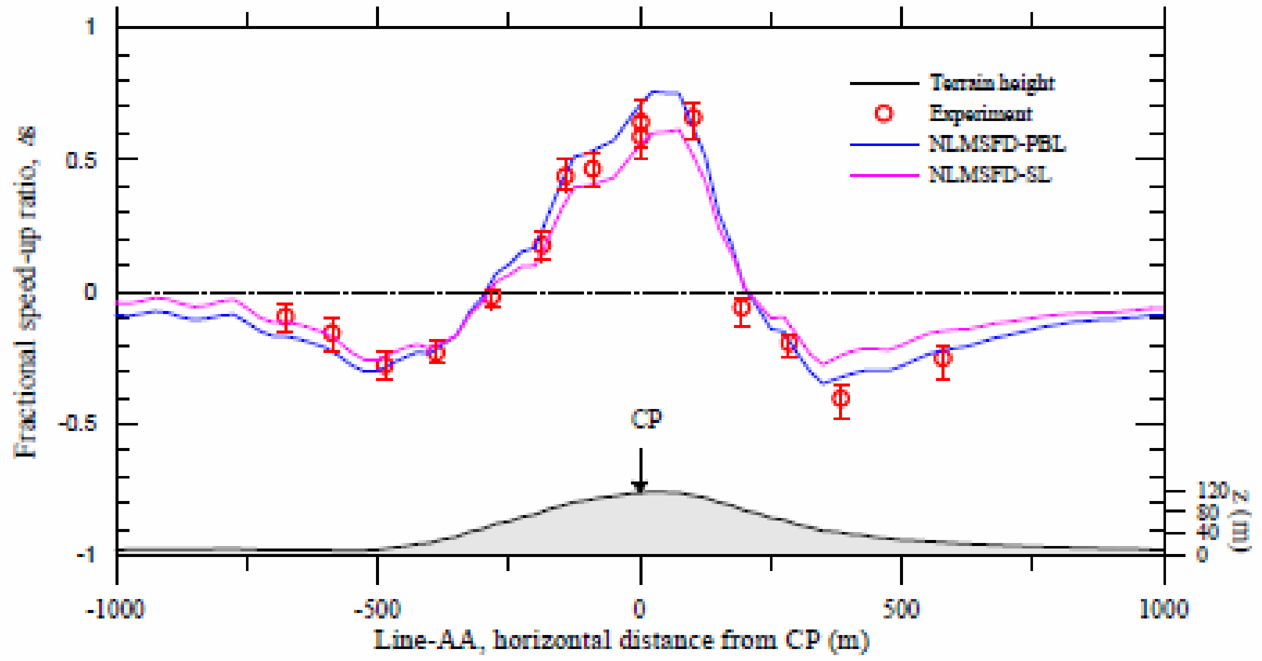


Figure 4. Fractional speed-up ratio, ΔS, for flow over Askervein hill at a height of 10 m above topography along line AA. Comparison of model results and experimental data.

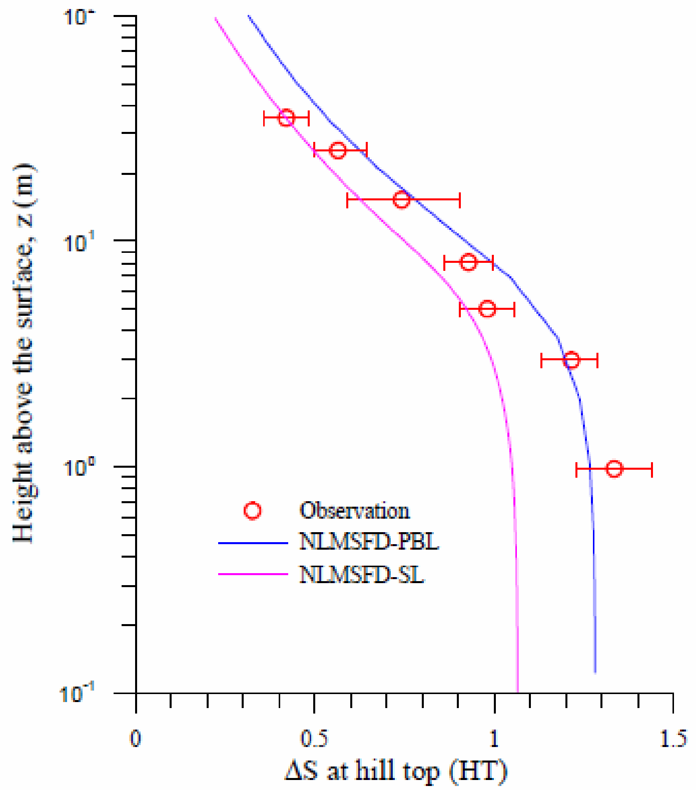
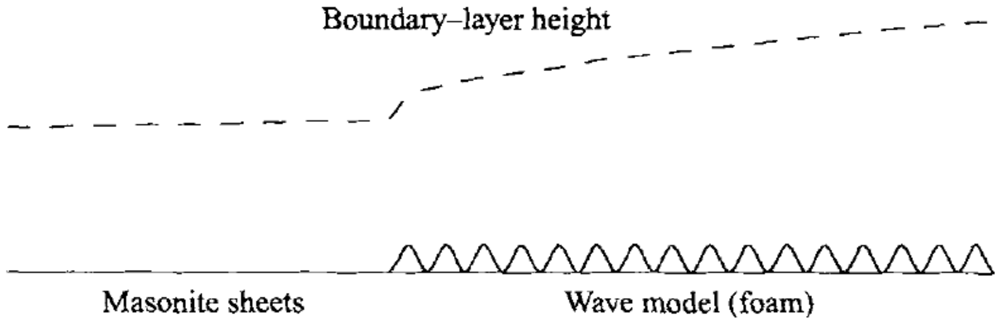


Figure 5. Vertical profile of fractional speed-up ratio, ΔS, at the hill top (point HT) of Askervein hill. Comparison of model results and experimental data.

Flow over wavy surfaces. Sand waves and water waves. (Gong, W., Taylor, P.A. and Dornbrack, A., 1996, Turbulent boundary-layer flow over fixed, aerodynamically rough, 2D sinusoidal waves, J. Fluid Mech., 312, 1-37. A wind tunnel study, and LES.



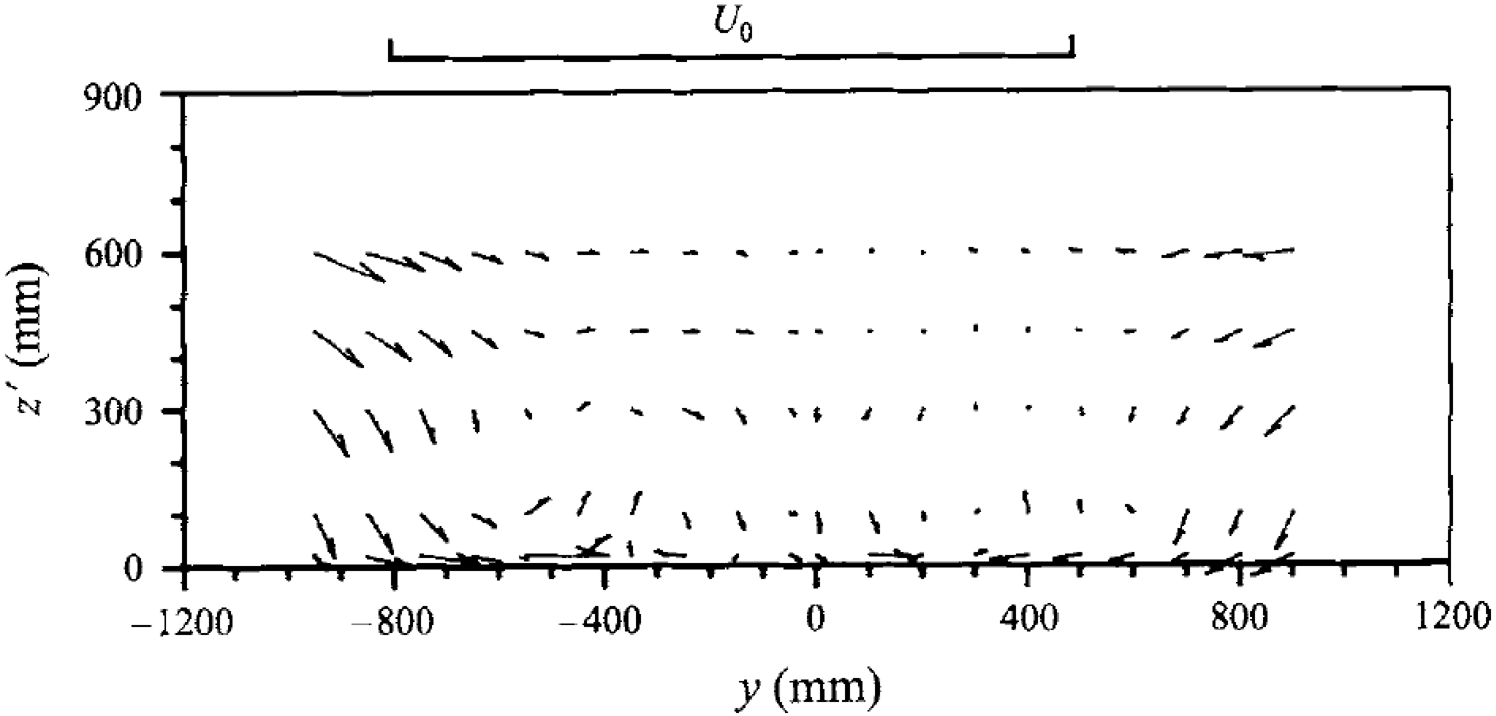


FIGURE 11. Vector plot of secondary flow (V, W) over the 12th crest (relatively smooth-surface case). Horizontal scale above the figure represents U_0 (approximately 10 m s^{-1}).

A flow situation I want to look at again. Env Canada wind tunnel closed down, study repeated in Western University BLWT lab. We tried in Canberra but wasn't satisfactory. Ocean waves are everywhere and this could increase fluxes. Link with Peter Gent's water wave modelling.

Conclusions.

Quite a lot of work over past 50 years. John’s reviews of flow over hills (1988, 1994, 2020) are thorough and full of excellent ideas. The 1988 chapter (Air Flow over Complex Terrain) and the “Commentary” by Niels Otto Jensen are well worth tracking down and reading.

Recent work covered in the 2020 review has tended to emphasize forest covered hills and canopy issues, and to apply LES models with maybe less RANS work – in the next sessions we can move on from bare hills and see some eddy resolving model results.

A few acknowlegements.

Askervein: Jim Salmon (contractor), Bob Mickle and Wes Kobelka from the Atmospheric Environment Service, Niels-Otto Jensen and Gunnar Dalsgaard from Riso, Axe1 Hoff and Gerd Tetzlti from University of Hannover, Tony Bowen from University of Canterbury, Nick Cook from the Building Research Establishment and Rob Johnson from ERA Technology Ltd.

Modelling: Southampton, Environment Canada and York: Kelvin Richards, Peter Gent, John Walmsley, Jim Salmon, Anton Beljaars, Dapeng Xu, Wensong Weng, Lucy Chan, Keith Ayotte.

Wind Tunnel: Wanmin Gong, Hans Teunissen, Keith Ayotte

MARINE STRATUS AND FOG - BOUNDARY LAYER MIXING AND A WATER DROPLET SINK.

Peter Taylor, ESSE, York University

A simple 1-D RANS model of the time evolution of the Planetary Boundary Layer is extended to include water vapor and cloud droplets plus transfers between them. Radiative fluxes and flux divergence are also included. An underlying ocean surface is treated as a source of water vapor, and as a sink for cloud or fog droplets. With a constant sea surface temperature and a steady wind, initially dry or relatively dry air will moisten, starting at the surface**. Turbulent boundary layer mixing will then lead towards a layer with well-mixed potential temperature (and so temperature decreasing with height) and well mixed water vapor mixing ratio. As a result the air will, sooner or later, become saturated at some level and stratus cloud will form.** If that air is later advected over colder water the air will cool and the base of the stratus cloud will lower. Fog may then extend down to the surface.

The liquid water mixing ratio is essentially zero at the surface but will increase with height to a maximum, typically in the lowest 100- 500 m height range, depending on conditions. Relative humidity is 100% throughout this layer and is what is typically observed, e.g. by G.I. Taylor in 1915. There are however no published observations that we know of, that provide the variations of liquid water mixing ratio with height in marine fog. I am hoping that analysis of data from the 2022 FATIMA program may provide these critical measurements.

Impacts of radiative fluxes and surface cooling. We look at a simple sample case. a) LW fluxes and no surface cooling; 24 h run. $\underline{U}_g$ = (20,0) ms^{-1}, initial dθ/dz = 1 K/km, initial surface temperature, 288 K, initial RH = 0.75 exp(-z/2000.0), No cloud in first 12 h. Note sharp cloud top. Max Ql at top.