Constant Flux Layers with Gravitational Settling: deposition to an underlying surface and links to fog.

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Abstract The turbulent boundary layer concepts of constant flux layers and surface roughness lengths are extended to include gravitational settling and surface deposition of fog or cloud droplets in neutrally and stably stratified atmospheric surface boundary layers.

Keywords Constant flux layers • Fog • Gravitational settling • Surface roughness

1. Introduction

Monin-Obukhov Similarity Theory (MOST) considers situations in steady state, horizontally homogeneous, turbulent atmospheric boundary layers where velocity and other variables can be simply dependent on height above the surface, z. In many situations vertical turbulent fluxes of, in particular momentum and heat, can be considered as approximately independent of z. With no sources or sinks of momentum or heat within these constant flux layers one can then use dimensional analysis to predict the form of the profiles. Garratt (1992, section 3.3) or Kaimal and Finnigan (1994) explain Monin-Obukhov similarity while Monin and Obukhov (1954) is a translation of the original Russian work. The simplest case is with neutral stratification where dimensional analysis can be used to infer that the velocity shear, dU/dz is simply proportional to u^*/z where the shear stress, assumed constant with height, is ρu^{*2}, with ρ as air density.

Integration of this relationship leads to

$$U(z) = (u^*/k) \ln(z/z_{0m}), \quad (1)$$

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with the roughness length for momentum, z_{0m}, being defined as the height at which a measured profile has $U = 0$ when plotted on a U vs ln z graph, and where k is the Karman constant with a generally accepted value of 0.4. Noting that z_{0m} values are generally small compared to measurement heights, and after a z_{0m} value has been established for the underlying surface, it is mathematically convenient to modify the relationship to

$$U = (u^*/k) \ln((z+z_{0m})/z_{0m}), \tag{2}$$

so that we have $U = 0$ on $z = 0$. In eddy viscosity terms this corresponds to

$$K_m = ku^*(z+z_{0m}) \tag{3}$$

In situations with constant, or near constant fluxes of heat and water vapour, similar logarithmic, or near logarithmic, MOST profiles and eddy diffusivities can be established, based on measured profiles involving z/L where L is the Obukhov length. For potential temperature and water vapour profiles these will involve additional "scalar" roughness lengths, z_{0h} and z_{0v}. Much has been written about roughness lengths and ratios between z_{0m} and z_{0h}, including Chapter 5 of Brutsaert (1982). For momentum transfers, form drag on roughness elements, sand grains, blades of grass, bushes, trees, buildings and water waves, can provide most of the drag on the surface and, except over water, z_{0m} is considered as a Reynolds number independent surface property. Water waves are wind speed dependent and z_{0m} needs to take this into account. For heat and water vapour the final transfers from air to the surface involve molecular diffusion and, as a result values of z_{0h}, z_{0v} are significantly lower than z_{0m}. We will introduce a separate roughness length for fog or cloud droplets, z_{0c}. There appears to be very little discussion of a roughness length for cloud droplets in the literature and there is a need for measurements of cloud droplet profiles in fog to establish appropriate values for modelers to work with.

Early fog models such as Brown and Roach (1976) or Barker (1977) assume the same eddy diffusivities for water vapour and cloud droplets, presumably with the same roughness lengths while models dealing with deposition of fog water to vegetation, such as Shuttleworth (1977), Lovett (1984) and Katata et al (2008) work in terms of deposition velocity (V_d) and resistance ($1/V_d$) rather than z_{0c}. Over forests, Lovett (1984) points out that there can be "turbulent transfer of cloud droplets to the canopy" and that, in windy conditions "inertial impaction is the dominant mechanism". However, the downward flux of cloud water may be due to both turbulent mixing and gravitational settling, as noted by Katata (2014), although he states that, " For relatively smooth surfaces such as bare soil and water ...... the mechanism of gravitational settling is assumed to be dominant."

We consider a situation with fog, or cloud being present and in contact with the lower boundary in, to start with, a neutrally stratified boundary-layer. Our hypothesis is that fog droplets will be deposited at the surface and that this can lead to an approximately constant flux layer situation, if the air in the constant flux layer is at 100% relative humidity. Fog droplets and the associated liquid water mixing ratio will then have a downward flux associated with a combination of gravitational settling of the droplets plus turbulent diffusion and removal due to collision with the surface and coalescence. It is expected that this process will be active over many surfaces and, in particular in marine fog situations over water. It is often claimed that turbulence can enhance the rate of collision and coalescence between droplets in clouds. For example, Franklin (2014) states "Although the effect of turbulence on cloud droplet collision–coalescence rates is yet to be quantified by observations, modelling studies have shown that turbulence can increase the collision rates of droplets by several times the purely gravitational rate." and cites several studies demonstrating this. We anticipate that the same effect will give enhanced deposition of fog droplets to water surfaces.

2. A simple model

Based on our hypotheses, we consider an idealized situation where the lowest layers of a horizontally homogeneous boundary-layer fog situation are at 100% relative humidity, are in a steady state and could be considered as having a constant downward flux of uniform size cloud droplets and associated liquid water mixing ratio with a sink at the water surface. The source would be above the constant flux layer where continued cooling of saturated air would create new droplets or allow others to grow. In reality many cloud micro-physics and radiation processes could be involved, but here we consider a simple model with just turbulent transfers and gravitational settling. One could then model the constant downward flux of fog, F_{Qc}, as

$$w_s Qc + ku^*(z + z_{0c})\, dQc/dz = F_{Qc} = u^* q_c^*, \qquad (3)$$

where w_s represents the gravitational settling velocity and u* is the friction velocity. The eddy diffusivity K_{qc} is assumed to be

$$K_{qc} = ku^*(z + z_{0c}), \qquad (4)$$

where z_{0c} is a roughness length for fog droplets with the assumption that $Qc = Qc_{surf}$ at $z = 0$.

Over a water surface we assume $Qc_{surf} = 0$. Initially we can assume a single drop size, with a single w_s and single z_{0c} but, provided we assume that individual drops retain their size and integrity as they pass through the constant flux layer at 100% relative humidity, one can apply these ideas to multiple size bins and combine the profiles of each to get $Qc(z)$ totals. Assuming constant values for z_{0c}, $u*$ and w_s one can then solve Eq (3), by integrating factor techniques, multiplying (3) by $(z+z_{0c})^{S-1}/(ku*)$ where $S = w_s/(ku*)$, to give,

$$(d/dz)[(z+z_{0s})^S Qc] = (q_c*/k)(z+z_{0c})^{S-1} \qquad (5)$$

and, with $Qc(0) = 0$ the solution is,

$$Qc(z) = (q_c*/(kS))\ [1- ((z+z_{0c})/z_{0c})^{-S}]. \qquad (6)$$

In terms of $\zeta = \ln((z+z_{0c})/z_{0c})$, we can write,

$$Qc(\zeta) = (q_c*/(kS))\ [1-e^{-S\zeta}]. \qquad (7)$$

These can be referred to as Constant Flux Layer with Gravitational Settling or CFLGS, profiles. In the limit as w_s and $S \rightarrow 0$, as $\zeta \rightarrow 0$, Eq (7) would give $Qc(\zeta) = (q_c*/k)\ \zeta$, a standard log profile.

3. Some profiles

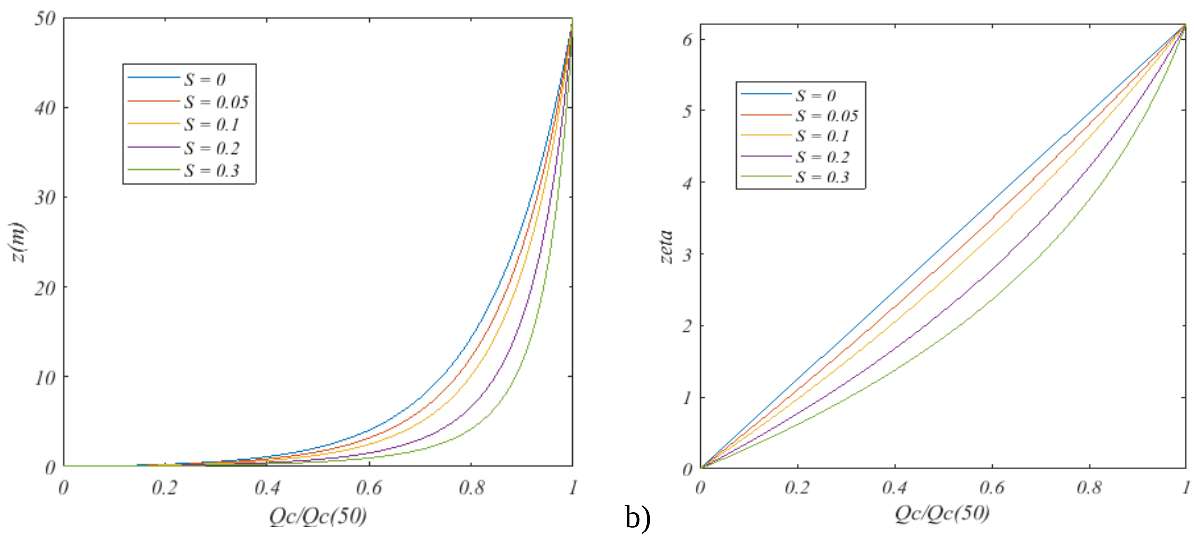
The expected values of w_s and $u*$ should be considered. Fog droplets have a range of sizes but most fall in the diameter, d, range 0-50 µm, often with bimodal distributions and peaks around 6 and 25 µm (see for example Isaac et al, 2020). Applying Stokes law, $w_s = gd^2(\rho_w-\rho)/(18\mu)$, with $\mu = \nu\rho_a$, where ν (15.06 x 10^{-6} m^2 s^{-1} at 20°C and standard pressure) is the kinematic viscosity of air. With air density, ρ_a (1.178 kg m^{-3}), water droplet density, ρ_w and acceleration due to gravity, g, for these peak sizes we get w_s values of 0.0011 and 0.0192 m s^{-1}. These terminal velocities are clearly small compared to wind speed but for the larger diameter droplets, where the bulk of the liquid water content, $LWC\ (=\rho_a Qc)$, is often measured, the terminal velocity corresponds to 69 m per hour and will represent a considerable removal rate in fog which may last several hours or days. The key parameter in our constant flux with gravitational settling model is $S = w_s/ku*$. In moderate winds over the ocean one might expect $u*$ values in the 0.2-0.5 m s^{-1} range, while in radiation fog in light winds over land it could be lower. The parameter, S will thus generally be in the range 0.006 to 0.3 over water but could be unlimited in calm conditions over land.

At low values of S gravitational settling will have little impact and Qc profiles will be approximately logarithmic. To illustrate this Fig. 1 shows Qc constant flux profiles with linear and log vertical axes and a range of S values. We have scaled Qc with a value at 50m. The main unknown is the value of z_{0c}. Here we use a relatively high value (0.1m) indicating efficient capture of water droplets by the water surface. Note that these calculations are for uniform sized droplets, with size related to $w_s^{0.5}$, or $S^{1/2}$ if u^* were fixed. Note that with high S $(= w_s/ku^*)$ values, maybe occurring with low u^* and minimal turbulence, the limiting case would be constant Qc down to $z = 0$ and a discontinuity to $Qc = 0$ at the surface. Calculations with $S = 1$ and 5 (not shown) confirm this. One way to look at the relative importance of gravitational settling for these uniform size droplets is to consider the relative contributions to the total downward flux of water droplets ($u^*q_c^*$). The gravitational contribution is simply w_sQc while the turbulent diffusion contribution is,

$$ku^*dQc/d\zeta = u^*q_c^*e^{-S\zeta}, \text{ where } \zeta = \ln\left((z+z_{0c})/z_{0c}\right) \qquad (8)$$

The ratios of turbulent transfer/total flux and gravitational settling/total flux then become

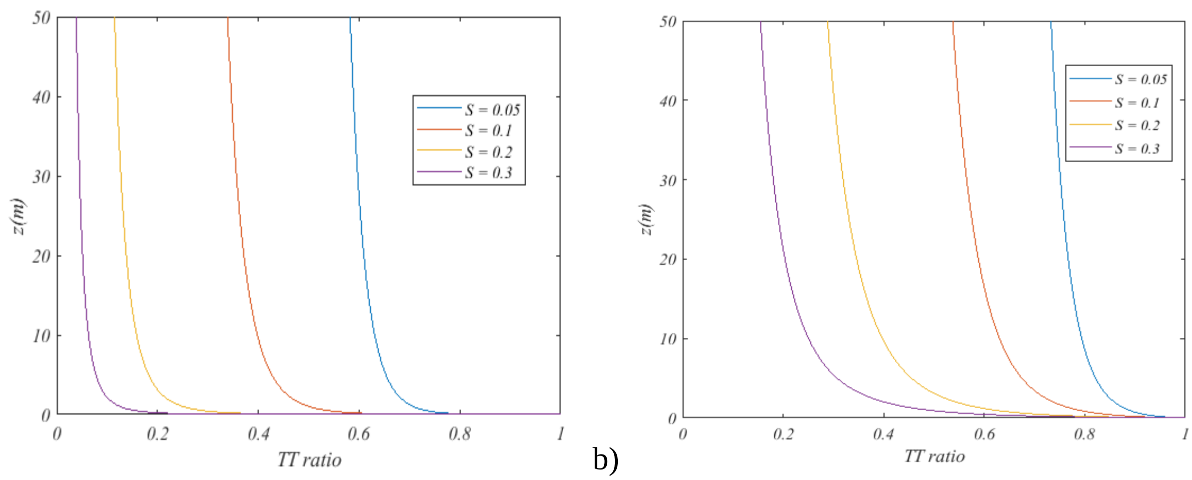
$$TT = e^{-S\zeta} \quad \text{and } GS = 1 - e^{-S\zeta} \qquad (9)$$



a) b)

Fig. 1 Qc profiles, scaled by 50 m value, from surface to $z = 50$ m in constant flux layers with gravitational settling and surface roughness length for water droplet removal, $z_{0c} = 0.1$ m. Linear (a) and logarithmic (b) height scales.

Noting that $\zeta = \ln((z+z_{0c})/z_{0c})$ we can see that these ratios depend on both z_{0c}, through the $z(\zeta)$ relationship, and S and will vary with z. Fig. 2 illustrates this. It is important to note that Fig. 2a is based on a relatively low estimate for z_{0c}, (0.001 m). If we increase it to z_{0c} = 0.1 m as in Fig. 1 then turbulent fluxes become more important. We can see that the TT ratio is formally 1 at the surface, where $Qc = 0$ so there is no gravitational component. For very large ζ the TT term would decay to 0 but this would be well above the constant flux layer approximation. At 50 m the value will depend on S and z_{0c}.



a) b)

Fig. 2 Variation of the Turbulent Transfer fraction of the total Qc flux and its variation with z and S. Note that these z values are based on a) z_{0c} = 0.001 m and b) z_{0c} = 0.1 m

4. Stable Stratification Case

Over land radiation fog often occurs at low wind speeds with stable stratification. For constant flux boundary layers in these circumstances MOST has, for velocity, $K_m = k(z+z_{0m})/\Phi_M(z/L)$ and

$$\Phi_M(\zeta) = 1 + \beta\,(z+z_{0m})/L : U = (u^*/k)\,(\ln((z + z_{0m})/z_{0m}) + \beta\, z/L). \qquad (10)$$

Observed profiles give $\beta = 5$ (Garratt 1992, p52). If we extend this idea to $K_{Qc} = k(z+z_{0c})/\Phi_{Qc}(z/L)$ with a similar form for Φ_{Qc} we need to solve,

$$w_s Qc + [ku^*(z + z_{0c})/\Phi_{Qc}(z/L)]\, dQc/dz = F_{Qc} = u^* q_c^*, \text{ or}$$

$$dQc/dz + S\{(1+\beta\,(z+z_{0c})/L)/(z+z_{0c})\}Qc = (q_c^*/k)(1+\beta(z+z_{0c})/L)/(z+z_{0c}); \quad S = w_s/(ku^*)$$

The Integrating Factor is $\exp(\int S(1/(z+z_{0c})+\beta/L)dz = (z+z_{0c})^S \exp(S\beta z/L)$ so that

$$d\,[(z+z_{0c})^S \exp(S\beta z/L)Qc]\,/dz = (q_c^*/k)(1+\beta(z+z_{0c})/L)\,(z+z_{0c})^{S-1} \exp(S\beta z/L) \qquad (11)$$

and we need to integrate the RHS. To do this it is convenient to let $\beta(z+z_{0c})/L = x$ and the integral that we need is of

$$(q_c{}^*/k)(L/\beta)^{S-1}exp(-Sx_0)\ \{(1+x)x^{S-1}exp(Sx)\} \qquad \text{where } x_0 = \beta z_{0c}/L$$

After some guidance and a few trials one can see that $d/dx\{x^S exp(Sx)\} = (Sx^{S-1} + Sx^S)exp(Sx)$ and the integral required is simply $F(x,S) = x^S exp(Sx)/S$. We can then evaluate $F(x,S)$ at $z = 0$, $x_0 = \beta z_{0c}/L$ and any z to allow us to plot Qc profiles.

With stable stratification and light winds the constant flux approximation would only apply to a relatively shallow layer so we normalize with $Qc(20m)$ in these cases.

Then if $Qc = 0$ at $z = 0$ we have

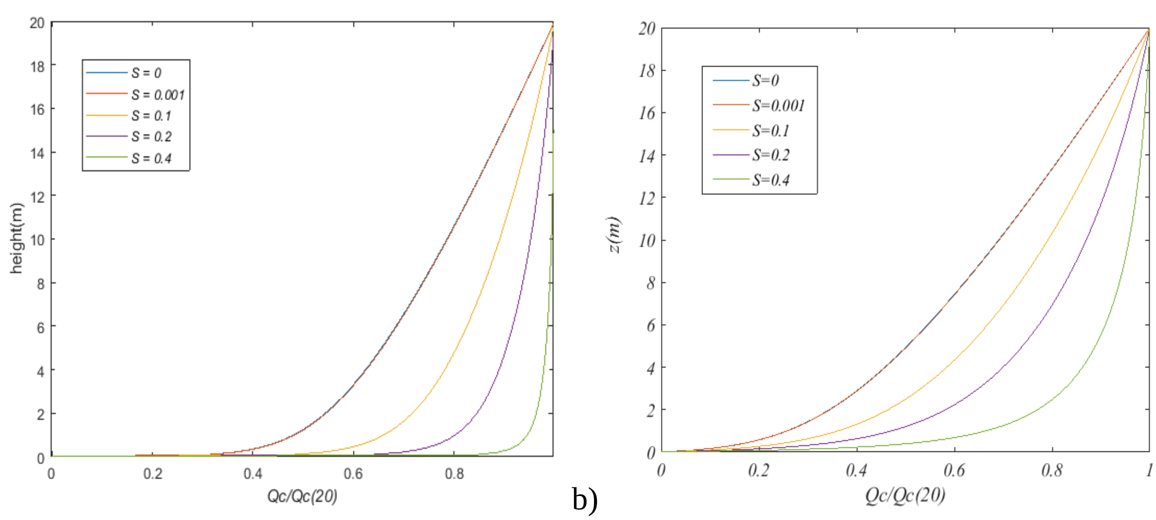
$$Qc(z) = [(q_c{}^*/k)(L/\beta)^{S-1}exp(-Sx_0)/\ ((z+z_{0c})^S\ exp(S\beta z/L))]\ [F(x,S) - F(x_0,S)], \qquad (12)$$

where $x = \beta(z+z_{0c})/L$ *and* $x_0 = \beta z_{0c}/L$ and

$$Qc(z)/Qc(20)=((z+z_{0c})^S exp(S\beta z/L))][F(x,S)-F(x_0,S)]/\{((20+z_{0c})^S exp(20S\beta/L))][F(20,S)- F(x_0,S)]\}$$

For $S = 0$, with no gravitational settling, the profile will be essentially the same as the velocity profile in (A1) above, i.e.

$$Qc(z) = (q_c{}^*/k)\ (ln\ ((z + z_{0c})/z_{0c}) + \beta z/L). \qquad (13)$$



a) b)

Fig 3. Profiles with stable stratification, $\Phi_{Qc}(\zeta) = 1 + \beta\ (z+z_{0c})/L$, $\beta = 5$, $L =$ 20m, S = 0 and 0.001 lines overlap as confirmation of our solution form. a) z_{0c} = 0.001m, b) z_{0c} = 0.1m.

In addition to z_{0c} and S the key parameter is the Obukhov length, $L = -\rho c_p u^{*3}\theta/(kgH)$, (>0). Neutral stratification corresponds to $L \to \infty$ while stable stratification relationships (H < 0) are generally limited to z/L < 1. If we are concerned with height ranges up to 10 or 20m then L = 10m would be considered as a very low value maybe with $u* \approx 0.13$ ms^{-1} and $H \approx -20$ Wm^{-2} as possible values. Figure 3 shows *Qc(z)/Qc(20m)* profiles in a typical case with z_{0c} = 0.001 and 0.1m, L = 20m and a range of S values. For large droplets, S = 0.4, Qc flux is dominated by gravitational settling while for smaller particles, S = 0, 0.1 and z_{0c} = 0.001m, turbulent mixing dominates the deposition process.

5. Implications and Potential Uses

The basic idea behind this analysis is that, in fog, cloud droplets can both fall toward the underlying surface through gravitational settling and be diffused towards the surface by turbulence. On contact they can coalesce with an underlying water surface or be removed on a hygroscopic surface. Some vegetation surfaces, such as grass, are hydrophobic but we assume that cloud droplets will still be retained and accumulate in drops on the surface so that cloud droplets will still be removed from the air. The modelling assumption used here is that *Qc(0)* = 0 on the surface although bouncing of droplets after impact is a possibility, even on water (Hallett and Christensen 1984). We argue that a water surface can be a significant sink for fog droplets.

One can use these ideas in modelling work, adapting the approach of Katata et al (2010, 2011) for radiation fog over forests, to deal with marine advection fog over the ocean. A critical unknown parameter in this work is the deposition velocity relating *Qc* at the lowest model level to the downward flux to the surface due to turbulent transfer. As in the analysis above, one can use a roughness length for cloud droplets, z_{0c}, as a tuning parameter. Katata et al (2010, 2011) also need a tuning parameter (their "removal efficiency") to establish a relationship between a deposition velocity and the wind speed at some level. The two can be related if there also a known momentum roughness length, z_{0m}, for the surface. Some models treat the diffusion of total water $Qt = Qv + Qc$, where Qv is water vapour mixing ratio and assume a common roughness length, z_{0q} for Qt and Qv. These values are usually very low << z_{0m} and based on molecular diffusion of water vapour to or from the surface. The surface boundary condition on Qt in fog is often based on 100% RH values at the surface implying that Qc = 0 there.

The bottom line is that this removal process needs to be taken account of in modelling and forecasting fog occurrence and development and we need to know more about it. Fog is an intermittent phenomenon so setting up 50-m or higher measurement masts in fog-prone locations will be good start. The PARISFOG study (Haeffelin et al 2000) included 30-m masts and LANFEX (Price et al 2018) used 50-m masts but the profile measurements did not include fog water, Qc, or visibility. In-situ vertical profiles of Qc were also missing in field programs like FRAM (Gultepe et al 2009) and C-Fog (Fernando et al 2021). C-Fog instrumentation at various sites included 10-m and 15-m masts and also a Radiometrics microwave radiometer for Qc profile measurements. These may well report interesting measurements but better vertical resolution is desirable. There were Qc measurements at two or more levels in earlier field measurements reported by Pinnick et al (1978) and Kunkel (1984) showing increases with height. More such measurements are needed with multiple measurement levels and measuring droplet size distributions, *Qc* or *LWC* values and ideally *Qc* fluxes, along with wind, turbulence, temperature and humidity profiles plus surface pressure and fluxes of momentum, heat and water vapour. Visibility measurements at multiple levels, 4 component radiation and air, aerosol and fog chemistry measurements could play an important role. From the modelling perspective we need values for z_{0c}, which will depend on surface type and probably on droplet diameter and on wind speed or friction velocity. Assuming that the lower layers, say 10-30 m of a deep fog layer, are in a steady, constant flux layer situation then the CFLGS profiles developed above could provide a framework for analysis of observations.

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References

Barker EH (1977) A maritime boundary-layer model for the prediction of fog. Boundary-Layer Meteorol 11:267-294

Brown R, Roach WT (1976) The physics of radiation fog. II. A numerical study Q. J. R. Meteorol. Soc 102:335–354

Brutsaert W (1982) Evaporation into the Atmosphere, Reidel, Dordrecht, Holland

Fernando H, Gultepe I, Dorman C, Pardyjak E, Wang Q, Hoch S. Richter D, Creegan E, Gabersek S, Bullock T, Hocut C, Chang R, Alappattu D, Dimitrova R, Flagg,D, Grachev A, Krishnamurthy R, Singh, D, Lozovatsky I, Fernand H (2021) C-FOG: Life of Coastal Fog. Bulletin of the American Meteorological Society 102: 10.1175/BAMS-D-19-0070.1

Franklin CN (2014) The effects of turbulent collision–coalescence on precipitation formation and precipitation-dynamical feedbacks in simulations of stratocumulus and shallow cumulus convection Atmos. Chem. Phys 14: 6557–6570

Garratt JR (1992) The atmospheric boundary layer, Cambridge, UK

Gultepe I, Pearson G, Milbrandt JA, Hansen B, Platnick S, Taylor P, Gordon M, Oakley JP, Cober SG (2009) The

Fog Remote Sensing and Modeling (FRAM) field project. Bull. Amer. Meteor. Soc 90:341–359

Hallett J Christensen L (1984) Splash and penetration of drops in water. Journal de Recherches Atmospheriques, 18: 225–242.

Haeffelin M, Bergot T, Elias T, Tardif R, Carrer D, Chazette P, Colomb M, Drobinski P, Dupont E, Dupont JC (2000) PARISFOG : Shedding new light on fog physical processes. Bull. Am. Meteorol. Soc 91:767–783.

Isaac GA, Bullock T, Beale J, Beale S (2020) Characterizing and Predicting Marine Fog Offshore Newfoundland and Labrador. Weather and Forecasting. 35:347-365

Kaimal JC, Finnigan JJ, (1994) Atmospheric Boundary Layer Flows, Oxford University Press, UK

Katata G, Nagai H, Wrzesinsky T, Klemm O, Eugster W, Burkard R (2008) Development of a land surface model including cloud water deposition on vegetation, Journal of Applied Meteorology and Climatology, 47, 2129-2146.

Katata G, Nagai H, Kajino M, Ueda H, Hozumi Y (2010), Numerical study of fog deposition on vegetation for atmosphere-land interactions in semi-arid and arid regions, Agric. For. Meteorol 150:340–353.

Katata G, Kajino M, Hiraki T, Aikawa M, Kobayashi T, Nagai H (2011) A method for simple and accurate estimation of fog deposition in a mountain forest using a meteorological model. Journal of Geophysical Research 116:D20102.

Katata G (2014) Fogwater deposition modeling for terrestrial ecosystems: A review of developments and measurements, J. Geophys. Res. Atmos 119: 8137–8159. doi:10.1002/2014JD021669

Kunkel A (1984) Parameterization of droplet terminal velocity and extinction coefficient in fog models. J. Climate Appl. Meteor 23:34–41.

Lovett GM (1984) Rates and mechanisms of cloud water deposition to a subalpine balsam fir forest. Atmos. Environ., 18: 361–371

Monin AS, Obukhov AM (1954) Basic laws of turbulent mixing in the surface layer of the atmosphere, Contrib. Geophys. Inst. Acad. Sci. USSR, 24 (151):163-187

Pinnick R, Hoihjelle DL, Fernandez G, Stenmark EB, Lindberg JD, Hoidale GB, Jennings SG (1978) Vertical structure in atmospheric fog and haze and its effect on visible and infrared extinction. J. Atmos. Sci 35:2020–2032.

Price JD, Lane S, Boutle IA, Smith DKE, Bergot T, Lac C, Duconge L, McGregor J, Kerr-Munslow A, Pickering M, Clark R (2018) LANFEX: A field and modeling study to improve our understanding and forecasting of radiation fog. Bull. Amer. Meteor. Soc 99:2061–2077, https://doi.org/10.1175/BAMS-D-16-0299.1.

Shuttleworth WJ (1977) The exchange of wind-driven fog and mist between vegetation and the atmosphere. Boundary-Layer Met 12:463-489.