

AN IMPLICIT CONTOUR MORPHING FRAMEWORK APPLIED TO COMPUTER-AIDED SEVERE WEATHER FORECASTING

Dominique Brunet, *Member, IEEE*, and David Sills

Abstract—We propose a contour morphing framework that allows large deformations and topological changes while guaranteeing non self-intersecting contours. Instead of explicitly matching contour points, the proposed algorithm combines an implicit contour representation with state-of-the-art deformable image registration. Synthetic and real-life examples from a meteorological application are presented to demonstrate the efficacy of the framework.

Index Terms—Contour morphing, non-rigid image registration, meteorological data, level-set methods.

I. INTRODUCTION

Contour morphing, also known as shape morphing, contour metamorphosis, contour blending, set interpolation, time interpolation, time-linking, in-betweening or volume reconstruction, consists of the following problem: Given two contours C_0 and C_1 , find a sequence of contours C_t with $0 \leq t \leq 1$ that interpolates between C_0 and C_1 . We seek a contour morphing algorithm that is the most physically natural and visually appealing. In particular, for each of the intermediate contours, (1) their smoothness should be preserved, and (2) no folding, i.e. self-intersection, should be allowed.

Contour morphing has seen applications in computer generated animation [1], computer graphics [2], volume reconstruction from cross-sections [3] and in fluorescent microscopy [4]. Contour morphing has to be distinguished from image morphing [5] as the former is concerned with shapes described by contours and the latter with image represented by pixel intensities. In this letter, we study the application of contour morphing to the computer-aided generation of severe weather threat areas. These areas, drawn for several "key frames", represent the best possible forecast based on numerical weather prediction model guidance and weather forecaster knowledge. The threat areas can then be interpolated in time to help produce user-specific space and time-referenced forecasts on demand (see [6]).

Most contour morphing techniques in the literature are limited to the time-interpolation between two simple polygons. The explicit framework can be summarized as follows:

D. Brunet and D. Sills are with the Cloud Physics and Severe Weather Research Section, Environment Canada, King City, ON, Canada. e-mails: dbrunet@uwaterloo.ca, David.Sills@ec.gc.ca.

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(1) features, such as vertex location, are extracted from the polygons, (2) registration between the features is performed, (3) a correspondence map (matching) between features is found, and (4) paths are drawn between the matched features. Methods have a varying degree of complexity and the best methods avoid problems such as folding (see [7]). Note that many papers instead focus on the related problem of contour matching (see [8], [9] and references therein), which essentially seeks to find the similarity between two contours. This explicit framework does not apply for morphing between meteorological contours, since multiple contours have to be morphed at the same time. In such cases, some contours might not have any match at all but instead may initiate or dissipate between two key frames. Also contours must be able to split or merge freely.

We propose an implicit framework for contour morphing, that is, to use the "level set" representation. This is an extension of Distance Field Interpolation (see [10]), as we perform image registration as an intermediate step. Works on shape interpolation via morphological operations (see [11], [12]) are also closely related. The difference being that our proposed framework allows non-rigid registration between signed distance functions instead of performing affine or rigid registration directly on sets. Another paper following a similar idea is [13], where the warping has to be guided manually from landmarks points, instead of being fully automatic.

Several papers combine an implicit representation with advanced image registration techniques, but they are limited to the problem of contour matching [14], [15], [16], [8]. Another related but slightly different problem is contour tracking, where a sequence of contours is sought from a sequence of images, thus combining image segmentation and contour morphing in one step. In [17] Bertalmio et al. solve this problem and call it morphing active contours.

The contour morphing framework we propose can also be compared with a direct surface interpolation by solving a variational problem [18], [10]. The drawback of such methods is that they require interpolation in a space two dimensions higher than the intrinsic dimension of the contour, which could be computationally prohibitive. Moreover, without describing the motion between contours as a vector field, it is harder to add and control extra constraints on the contour motion.

II. METHODOLOGY

As input, the algorithm takes hand-drawn or automatically extracted contours. A contour can be represented as a set of polygons, each described as a set of coordinates describing their outer boundary, and if necessary, their inner boundaries. We assume a valid representation, i.e. that the polygons are simple and non-overlapping. In this case, an alternative representation would be a binary image describing whether a pixel location is inside or outside any of the polygons. We skip any discussion on the pre-processing and post-processing of contours and on the conversion between the two representations and instead focus on the contour morphing represented as binary images.

The contour morphing algorithm is divided into four main steps: (i) Distance transform, (ii) Registration, (iii) Warping, and (iv) Blending. Note that the first three steps are used with contour matching, the last three steps with image morphing, and the first and last step with Distance Field Interpolation.

A. Distance Transform

Let (X, d) be a metric space. For example, take $X = \mathbf{R}^2$ and d the Euclidean distance. The distance transform of $A \subset X$ at $x \in X$ is

$$\text{DT}(x, A) := \inf_{a \in A} d(x, a) \text{ for } x \in X.$$

Note that the distance transform is zero if $x \in A$. Let $A^C := X \setminus A$ be the complement of A . The signed distance transform of $A \subset X$ at $x \in X$ is

$$\text{SDT}(x, A) = \begin{cases} \text{DT}(x, A) & \text{if } x \in A^C, \\ -\text{DT}(x, A^C) & \text{if } x \in A. \end{cases}$$

The signed distance transform is zero on the boundary of A . This gives an implicit representation of a contour as the zero level set of a function.

The distance transform is commonly used not only for contour matching and morphing but also for contour evolution, image segmentation, morphological operations and to define the Hausdorff metric on sets. A limitation of the distance transform is that it is not differentiable at locations where the closest point is not unique.

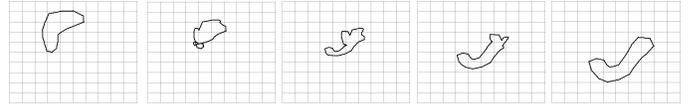
B. Image Registration

Image registration is a well-studied image processing problem [19]. We take advantage of recent advances and choose an algorithm that allows deformable matching while also being efficient and easy to implement. Demons-type algorithms [20] are selected for this purpose, but obviously any good registration algorithm can be inserted into this framework. Following the work of Vercauteren et al. [21], we can describe the demons algorithm as one possible numerical solution of the following optimization problem:

$$\arg \min_u \|f(x+u) - g(x)\|_2^2 + \lambda \|\nabla u\|_2^2, \quad (1)$$

where u is the displacement field. The parameter λ controls the trade-off between similarity and smoothness of displacement.

Fréchet distance minimization:



Distance Field Interpolation:



Proposed:

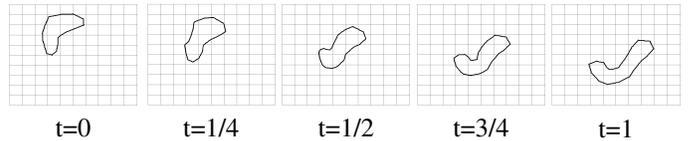


Fig. 1. Contour morphing between two concave shapes using three different methods. In the Fréchet distance minimization, vertices follow straight paths leading to the self-intersection of the contour. In the Distance Field Interpolation, the absence of overlapping features lead to the disappearance and the re-appearance of the shape. The proposed combination of distance transform with an image registration avoids both problems.

C. Image Warping and Blending

For the warping and blending parts, we choose the simplest option which is to consider straight paths for each location and linear interpolation between the intensities. The implicit contour morphing framework could allow more advanced geometric motion and intensity interpolation, but these options are not considered here.

After calculating the forward u and inverse displacement field u^{-1} between the images f_0 and f_1 , the morphed images are computed via

$$f_t(x) = (1-t)f_0(x+ut) + tf_1(x+u^{-1}(1-t)), \quad (2)$$

for $0 \leq t \leq 1$. The morphed contours C_t are the set of points such that $f_t(x) \leq 0$ for each $0 \leq t \leq 1$.

III. EXPERIMENTAL RESULTS

A. Contour Morphing Toy Examples

Some features of the proposed algorithm are illustrated with toy examples: no folding of contours (Fig. 1), split and merge (Fig. 2), and handling of topology changes (Fig. 3). Each example is generated by hand, drawing two frames containing one or multiple polygons and applying the algorithm without any tweaking or tuning of any parameter. The proposed algorithm is compared with explicit contour morphing (e.g. Fréchet distance minimization [22]), Distance Field Interpolation and direct image registration (e.g. Thirion's demons).

B. Meteorological Example

As an application of the proposed contour morphing algorithm, we study the problem of time-linking meteorological objects. Using expertise and varied sources of information such as point observations, radar and satellite imagery, numerical weather prediction and numerically computed storm

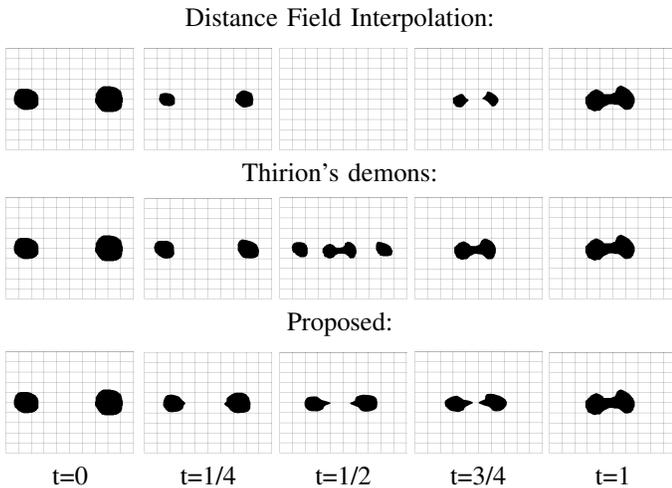


Fig. 2. Contour morphing with merging. Two disks are morphed into a peanut shape using three different methods. Once again, the shapes disappear and reappear with the Distance Field Interpolation. Direct image registration via Thirion's demons leads to a duplication of the shapes. Only the proposed algorithm is able to successfully handle this case.

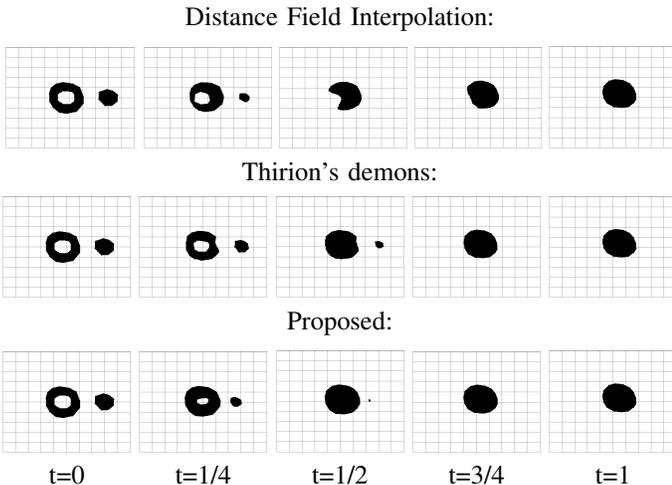


Fig. 3. Contour morphing with topology changes. A ring and a small disk are morphed onto a bigger disk using three different methods. With the Distance Field Interpolation, the interior of the ring does not move, leading to an unnatural interpolation. With direct image registration (Thirion's demons), the interior of the ring is suddenly filled without a smooth transition. Combining the distance transform with the image registration leads to the most physically natural and visually appealing result.

indices, a severe weather forecaster draws thunderstorm threat areas at present and future times within 48 hours. Since this work is labour intensive, we want to take advantage of the strong correlation between successive frames to automatically interpolate contours from a reduced set of key frames.

Figure 4 shows an example of the output of the algorithm at every hour for two key frames separated by six hours. A map of the north-eastern North America has been underlain so that the storm threat locations can be geo-referenced.

As a further illustration, we can visually compare algorithm output from T+3h in Fig. 4 with forecaster-drawn areas that were valid at that time (Fig. 5). Although the proposed algorithm leads to smoother contours than the interpolated

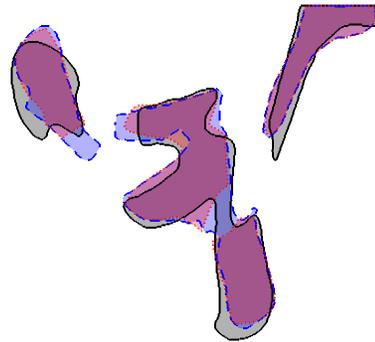


Fig. 5. Comparison of contours at T+3h obtained from Distance Field Interpolation (red with dotted boundary) and proposed algorithm (blue with dashed boundary) with manually drawn ground truth (black with full boundary). Although the proposed algorithm leads to more visually pleasing contours, the overall agreement with the ground truth is similar for both methods.

distance transform, neither of the two algorithms appear totally satisfying when comparing with the ground truth. However, when comparing the original contours (Fig. 4 at T+0h and T+6h) with the interpolated contours as well as with the ground truth as shown in Fig. 5, it becomes evident that some information in the forecaster-drawn contours at T+3h was not present in the original contours, and therefore could not be expected to be reproduced via interpolation alone.

C. Objective Validation

A number of sequences of three successive frames were extracted from contours drawn by a forecaster so that all the frames contain at least one severe weather threat area (not shown). Four sequences with a time interval of three hours and eight sequences with a time interval of six hours were thus obtained. Interpolation between the first and the last frame of each sequence (six or twelve hours difference) was carried out using either the Distance Field Interpolation or the proposed algorithm. As with the previous section, the middle frame of each sequence was kept as the ground truth. This was then compared with the interpolated contours using the symmetric difference and the Hausdorff distance. The results are summarized in Table I.

A two-sided t-test at the 5% level reveals that there is not a statistically significant difference between the scores obtained by the two algorithms for either six or twelve hours interpolation. However (but not surprisingly), there is a statistically significant difference between the scores obtained by the same algorithm but at different time intervals.

IV. DISCUSSION AND CONCLUSION

A flexible and robust implicit contour morphing framework has been proposed. A combination of a distance transform with Thirion's demons registration has been studied as a prototypical example. The use of level sets for representing contours has the advantage of allowing seamless splitting and merging of contours as well as allowing topological changes and avoiding any folding of the contours. The application of this framework for interpolating key frames of severe weather



Fig. 4. Time-linking of severe weather threat area forecasts over six hours in the north-eastern North America. The areas at T+0h and T+6h were drawn by a forecaster, while the areas in between were automatically drawn by the proposed algorithm. The contour morphing appears to be smooth and natural.

	Symmetric Difference (% area)		Hausdorff distance	
	$\Delta T = 6h$	$\Delta T = 12h$	$\Delta T = 6h$	$\Delta T = 12h$
DFI	7.8 ± 4.5	12.1 ± 2.5	43.2 ± 27.5	100.9 ± 48.0
proposed	8.2 ± 4.4	11.2 ± 2.1	45.2 ± 27.2	92.8 ± 45.5

TABLE I

OBJECTIVE SCORES (MEAN \pm STANDARD DEVIATION) FOR CONTOUR MORPHING OF METEOROLOGICAL OBJECTS WITH TIME DIFFERENCE $\Delta T = 6H$ AND $\Delta T = 12H$ FOR THE DISTANCE FIELD INTERPOLATION (DFI) AND THE PROPOSED METHOD.

forecasts appears to be particularly useful when the time difference between frames is the greatest.

Examples clearly demonstrated that both the distance transform step and the image registration step are necessary for large deformations of contours. This can be explained by the fact that the distance transform leads to an implicit representation of contours that will not suffer from the aperture problem at the image registration step.

However, the computation time required for image registration is generally greater than that for explicit contour matching, so this trade-off has to be considered depending on the application. Nevertheless, a Matlab implementation of the proposed algorithm ran in seconds for toy examples to minutes for more complicated cases.

Another possible limitation of this framework is that the implicit contour representation might allow splitting, merging and other topological changes more often than desired, since there are no constraints preventing these from happening. Also, the linear blending might not optimally represent the highly non-linear nature of severe weather. Other kinds of blending, for example taking for account the circadian cycle, could also be investigated.

Extension of the algorithm to multiple level sets, for multivariate data and for smooth morphing over several frames is planned in the future. In particular, it is desired that the algorithm be able to morph two or more frames containing severe thunderstorm threat areas with sub-areas representing different probability levels for both thunderstorm occurrence and severity.

The potential for powerful contour morphing algorithms has been demonstrated in this letter. It is our hope that this new application for meteorological forecasting will provide the impetus to further develop contour morphing algorithms.

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