Laboratory Exercise #5: Properties of a Transiting Extrasolar Planet

NAME ______________________________ **ID#** ____________________

DATE _______________________ **LAB SECTION#** _________________

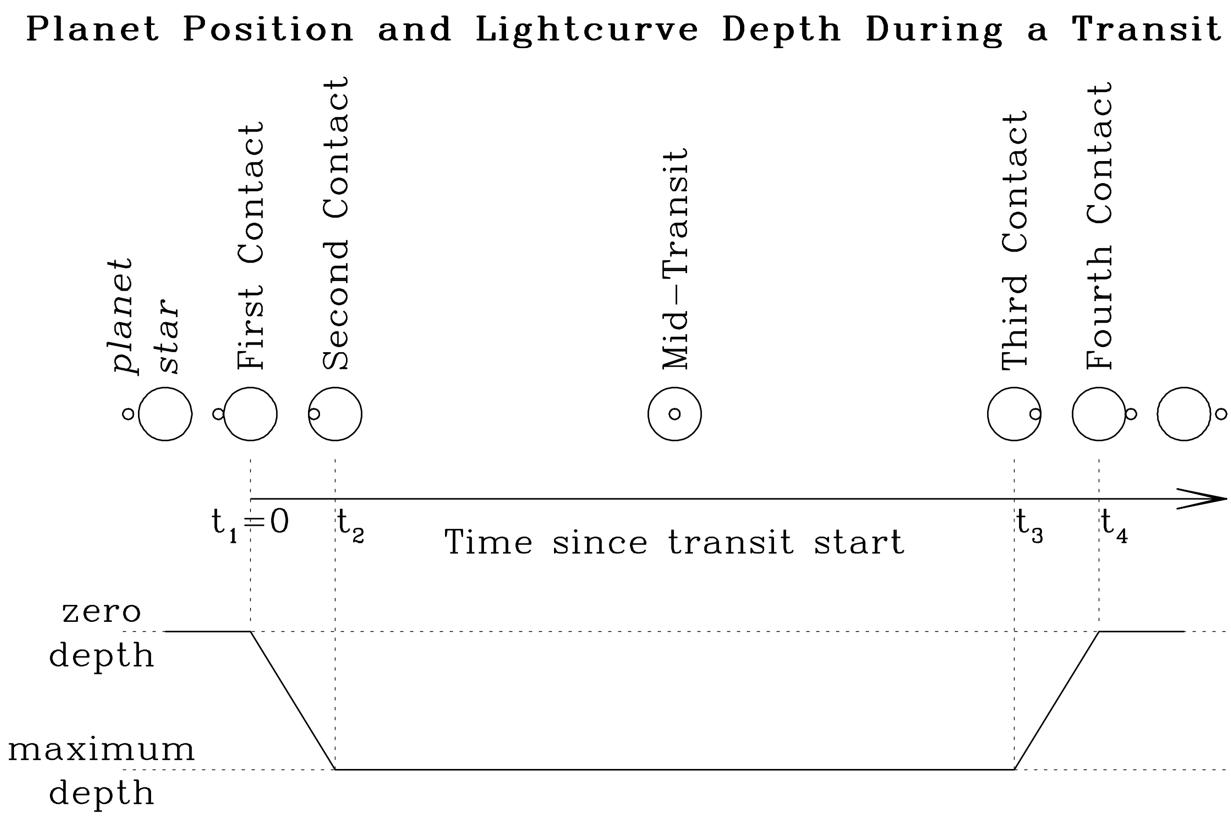
Write your answers on each page of this lab and hand the entire lab in.

In this lab you will study an extrasolar planet in a circular orbit around a star with the same mass, radius and surface temperature as our Sun. You will use the lightcurve of one transit of the planet to determine the planet's radius, mass, density, distance from its star, and surface gravity. Along the way, you will compare your planet to planets in our solar system.

Below we have illustrated what happens during a transit: a planet (small circle) moves between us and the star it orbits (big circle). As time goes on after a transit starts, the lightcurve of the star (bottom) shows that an increasing amount of the light from the star is blocked until a maximum depth is reached (time t_2, Second Contact). The planet begins moving out from in front of the star at time t_3 (Third Contact), and the transit ends at time t_4.

1) Your lab coordinator will give you a unique chart. That chart is a simulated lightcurve of the end of one transit of a unique planet. Near the bottom of the chart, the planet is given a number.

Record the number of your planet here: _________



Radius of Your Planet

The maximum depth of a planet's transit gives the planet's area as a fraction of the star's area. Because the area of a circle is $\pi(\text{Radius})^2$ (we assume star and planet are both perfectly round), the radius of the planet as a fraction of the star's radius is the square root of the depth:

$$\text{Depth} = \frac{\text{Planet Area}}{\text{Star Area}} = \frac{\pi(\text{Planet Radius})^2}{\pi(\text{Star Radius})^2} \quad \longrightarrow \quad \frac{\text{Planet Radius}}{\text{Star Radius}} = \sqrt{\text{Depth}}$$

2) Measure the maximum depth of your planet's transit on the chart you were given. Note that the numbers increase DOWNWARDS on the graph. You should measure the depth to within at least 0.0001 (at least four numbers to the right of the decimal point, but no more than five).

$$\text{Maximum Depth} \; = \; \_ \; . \; \_ \; \_ \; \_ \; \_ \; \_ \qquad \text{(A)}$$

3) Take the square root of the depth to find your Planet's radius relative to the Star's radius. Write down five numbers to the right of the decimal point.

$$\sqrt{\text{Maximum Depth}} = \; \_ \; . \; \_ \; \_ \; \_ \; \_ \; \_ \qquad \text{(B)}$$

4) It's easier to understand the planet's size relative to Earth's size. Because the star is identical to the Sun, it has a radius equal to 109 times the radius of the Earth. Therefore we just have to multiply by 109 to find the planet's radius relative to the Earth's radius:

$$\text{Planet's Radius relative to Earth's Radius} = 109 \times \sqrt{\text{Maximum Depth}} = \_ \; \_ \; . \; \_ \; \_ \qquad \text{(C)}$$

(In your answer you should have only 1 or 2 numbers to the *left* of the decimal point and should keep 2 numbers to the right of the decimal point.)

5) Use Table 1 to answer the following question comparing your planet's radius to the radii of some planets in our Solar System.

My planet's radius is in between that of the planets ____________ and ____________.
(For example, if your planet's radius relative to Earth's was 0.7, the correct answer is "between Mercury and Venus", because those are the planets with radii closest to yours.)

Table 1: Some Useful Properties of Some Planets in our Solar System

Planet	Radius / Earth's	Mass / Earth's	Density in grams/cm^3	Semi-major Axis / Star's Radius	Surface Gravity / Earth's	Cloud-top Temperature
Mercury	0.38	0.055	5.43	83.3	0.38	449 K (176 C)
Venus	0.95	0.815	5.25	155.6	0.91	329 K (56 C)
Earth	1.00	1	5.52	215.2	1	279 K (6 C)
Jupiter	11.19	317.9	1.33	1120.	2.36	123 K (-150 C)
Saturn	9.46	95.18	0.70	2053.	0.92	91 K (-183 C)
Neptune	3.81	17.13	1.64	6469.	1.14	51 K (-222 C)

Distance of Your Planet from its Star

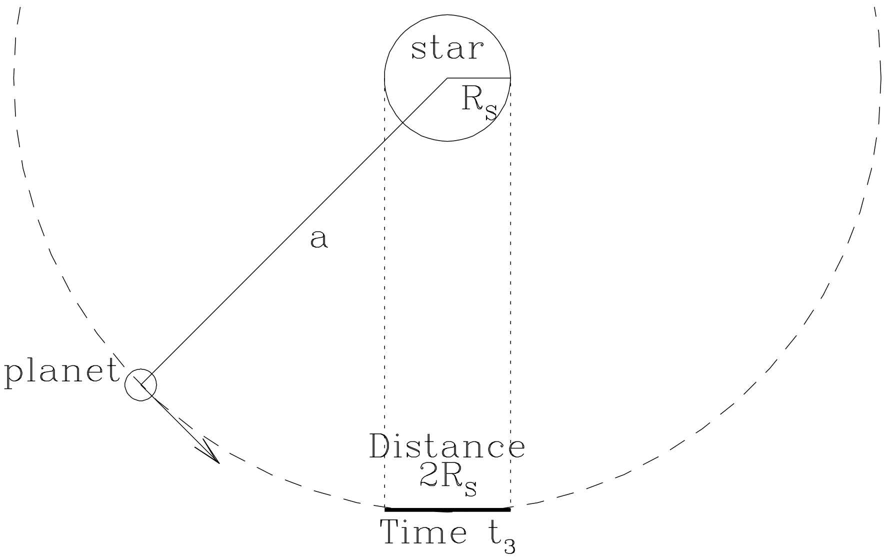


Figure 1: A planet orbiting its star in a circular orbit of semi-major axis a, looking top-down onto the orbit. Earth is far off in the direction of the bottom of the page. When the planet is between the star and Earth, a transit occurs.

Now that you have measured the length of a transit for your planet, you can calculate your planet's distance from its star, a, in units of the star's radius.

The distance around a circular orbit is 2π times the radius of the orbit. So your planet travels a distance $2\pi a$ in time P_{days} (one orbital period), which means its velocity is $v = 2\pi a/P_{days}$. Your planet also travels a distance $2R_s$ in time t_3 (during one transit), which means its velocity also equals $v = 2R_s/t_3$. Those two expressions for the velocity must equal each other, which happens if:

$$\text{Planet's distance from star relative to the star's radius} = 7.64 \times \frac{P_{days}}{t_3}$$

where t_3 is measured in hours.

6) Look at the bottom of your individual chart and find the number of days between each transit; that is the period P_{days} of your planet's orbit, measured in Earth days. Multiply P_{days} by 7.64 and call the result D.

$$D = 7.64 \times P_{days} = \underline{\qquad\qquad} \qquad \text{(D)}$$

7) Measure t_3, the time of Third Contact (end of maximum depth) on your transit chart (bottom panel of your handout), in hours:

$$t_3 = \underline{\qquad\qquad}$$

Divide D by t_3 to get the semi-major axis of your planet's orbit around its star, in units of the star's radius:

$$E = \frac{D}{t_3} = \frac{\qquad\qquad}{} = \underline{\qquad\qquad} \qquad \text{(E)}$$

8) According to Table 1, are there any planets in our solar system with such a small distance from the Sun? If so, what planet(s)?

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Mass of Your Planet

To measure the mass of your planet requires measuring the motion of the star around which it orbits. A planet in a circular orbit moves in a circle around the center of mass of the star-planet system, at the same time as the star moves in a much smaller circle around the same point. The planet and star are on exactly opposite sides of the center of mass at all times, which means that the star and planet velocities are related by:

$$v_p = v_s M_s / M_p$$

But we also know that the planet travels a distance $2\pi a$ in one orbital period P_y (measured in years), so the planet's velocity also equals:

$$v_p = 2\pi a / P_y$$

Those two equations for v_p must equal each other, and if we combine them we get an expression for the planet's mass which we will use below:

$$\underline{M_p = v_s P_y M_s / 2\pi a}$$

9) The top panel of your handout shows the radial velocity of your planet's star, starting from the start of a transit. As the planet transits and moves away from us to circle around the star, the star moves towards us (positive radial velocity). The star's velocity towards us reaches a maximum and then drops back to zero and goes negative as the star begins to move away from us while the planet moves back towards us. The star reaches a minimum velocity and then after one orbital period the star's velocity is zero while another transit occurs.

Measure v_s, the **maximum** radial velocity of the star in the top panel of your handout:

$v_s =$ ________________________ meters per second (m/s) (F)

10) Multiply that number by P_{days} and call the result H.

$H = v_s \times P_{days} =$ ________________________ (H)

11) Divide H by E, your planet's orbital radius in units of the star's radius, and call the result J.

$J = \frac{H}{E} =$ ________________________ (J)

12) Multiply J by 6.605 and call the result M; that is your planet's mass relative to Earth's mass.

$L = J \times 6.605 = M = \frac{\text{Planet Mass}}{\text{Earth Mass}} =$ _ _ _ . _ _ (M)

13) Use the answer to question 12 and Table 1 to answer the following question.

My planet's mass is in between that of the planets ________________ and ________________.

Density of Your Planet

Now that you know your planet's radius (size) and mass, you can determine its density (mass divided by volume) and compare it to high-density terrestrial planets and low-density giant planets in our solar system.

14) Cube your Planet's Radius relative to Earth (C) and call the result N:

$$C^3 = C \times C \times C = N = \_\_\_\_\_\_\_\_\_\_\_\_ \qquad (N)$$

15) Divide your planet's mass M by N and call the result Q:

$$\frac{M}{N} = Q = \_\_\_\_\_\_\_\_\_\_\_\_ \qquad (Q)$$

16) Multiply Q by 5.5 to get your planet's density R in grams per cubic centimeter (cm^3):

$$Q \times 5.5 = R = \_\_\_\_\_\_\_\_\_\_\_\_ \qquad (R)$$

17) According to Table 1, your planet's density R is most similar to the density of (circle one):

(*A*) • A terrestrial planet like Mercury, Venus or Earth

(*B*) • A normal gas giant planet like Jupiter

(*C*) • A low-density gas giant planet like Saturn

(*D*) • An ice giant planet like Neptune

How Many Times Your Earth Weight Is Your Weight on Your Planet?

What you feel as weight is the force of Earth's gravity pulling on your body. The more mass you have, the more Earth pulls on you. The force of any planet's gravity per unit mass (one g) is proportional to the planet's mass divided by its radius squared: $g = GM/R^2$. If you were to stand on another planet (or on a blimp in its outer atmosphere, if it doesn't have a solid surface), you would feel a different weight. Let's see how much more you would weigh on your planet.

18) Square your Planet's Radius (C) relative to Earth and call the result S:

$$C^2 = C \times C = S = \_\_\_\_\_\_\_\_\_\_\_\_ \qquad (S)$$

19) Divide your planet's mass M by S; the result – call it W – is your planet's surface gravity relative to Earth's:

$$\frac{M}{S} = W = \_\_\_\_\_\_\_\_\_\_\_\_ \qquad (W)$$

In other words, on your planet you would weigh W times as much as on Earth.

20) Use Table 1 to compare your planet's surface gravity to that of planets in our solar system:

The surface gravity of my planet is greater than that of ________________ but less than that of ________________. (If there is no planet in Table 1 with a larger or smaller surface gravity than your planet's, put 'no planet in Table 1' in the appropriate blank.)

(Continue to the next page for the last question)

Summary of Your Planet

21) In one sentence, give your planet's properties as compared to planets in our solar system. For example: 'My planet orbits at the distance of Venus but is bigger than Neptune with a mass close to Jupiter's, giving it a terrestrial planet's density and a surface gravity four times Earth's.'

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Extra Credit
Cloud-Top Temperature of Your Planet

The cloud-top temperature of a fast-rotating planet is determined by the balance between how much heat the planet absorbs from the star and how much heat it emits into space. The size of the planet does not matter: a bigger planet absorbs more heat but also emits more heat. What does matter is how close the planet is to the Sun, and how much heat from the Sun it reflects rather than absorbs.

If we assume that the planet absorbs all the heat that hits it, we can estimate the temperature at the top of its atmosphere. (If the planet has no atmosphere, the cloud-top temperature will just be the surface temperature. But if the planet does have an atmosphere, the greenhouse effect can increase the surface temperature above the cloud-top temperature.)

The cloud-top temperature of your planet, or any planet orbiting a star like the Sun, is

$$\text{Cloud}-\text{top Temperature} = \frac{4100 \text{ Kelvin}}{\sqrt{\text{Planet's orbital radius a, relative to the star's radius}}}$$

22) [Extra Credit] Calculate your planet's cloud-top temperature in Kelvin:

$$T = \frac{4100 \text{ K}}{\sqrt{E}} = \_\_\_\_\_\_\_\_ \text{ K} \qquad \text{(T)}$$

23) [Extra Credit] Convert your planet's cloud-top temperature to degrees Celsius:

$$U = T - 273 = \_\_\_\_\_\_\_\_ \text{ C} \qquad \text{(U)}$$

24) [Extra Credit] Use Table 1 to compare your planet's cloud-top temperature to that of planets in our solar system. (If there is no planet in Table 1 with a lower or higher cloud-top temperature than your planet's, put 'no planet in Table 1' in the appropriate blank.)

The cloud-top temperature of my planet is greater than that of ____________ but less than that of ____________.