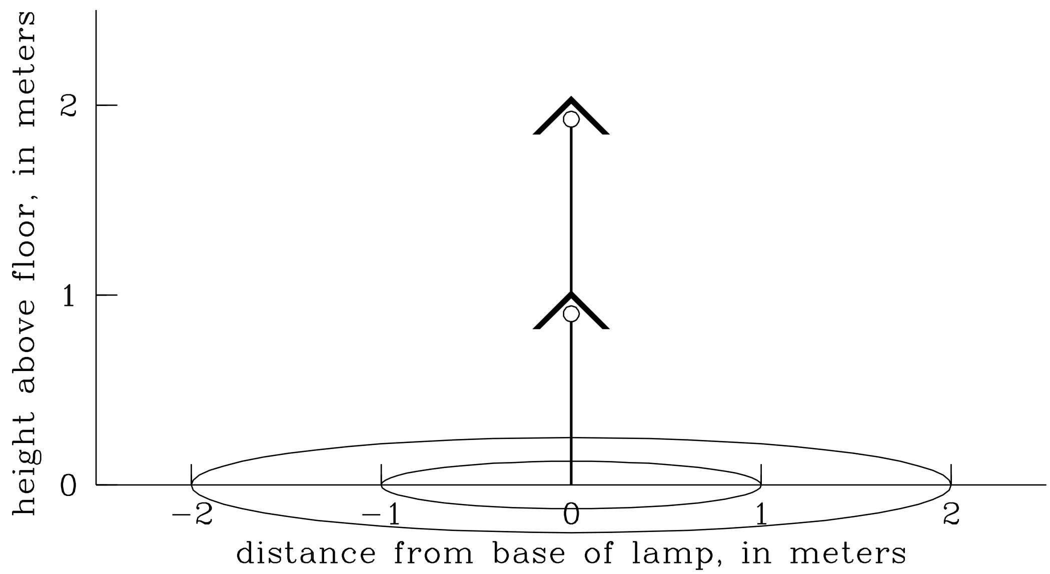
Luminosity, Brightness, and the Inverse Square Law

Luminosity is the total amount of energy per second given off by some object, such as a person, a lightbulb, or a star. Energy per second is usually measured in Watts (for example, a 100-Watt light bulb uses twice as much energy per second as a 50-Watt bulb).

Brightness is the amount of energy per second **per unit area** received from some object. Two objects from which the same light energy per second per square centimeter reaches your eye will look equally bright to you. Brightness can be measured in Watts per area, such as Watts per square meter.

Imagine a lamp with two lightbulbs, one at the top (2 meters high) and one half-way up (1 meter high). Each lightbulb has an opaque shade that directs all the light from the lightbulb downwards as shown. Also shown are two circles on the floor around the lamp.



1) Suppose the lightbulb 1 meter above the floor is turned on. How far around the lamp in every direction does the light from that lightbulb hit the floor directly?

2) How many square meters of floor area are lit up by the lightbulb located 1 meter off the floor? Note that the area of a circle of distance r from center to edge is πr^2, where π is a number close to 3.14 and where $r^2 = r \times r$.

3) Suppose the lightbulb puts out 314 Watts of light energy. On average, how many Watts per square meter of light energy does the floor receive from this light bulb? Assume a perfect mirror inside the shade, so that all the light from the bulb reaches the floor.

4a) When the lightbulb 2 meters off the floor is turned on, how far around the lamp in every direction is the floor lit up? How many square meters of floor area are lit up in this case?

4b) How many Watts per square meter of light energy does that floor area receive, on average?

4c) How much less bright will the floor look (in a photograph, for example) as compared to when the lightbulb is 1 meter off the floor?

Stars

As the luminosity from a star travels out into space at the speed of light, it spreads out over an ever larger area. For example, a light-year is the distance travelled by light in a year. One year from now, the light emitted by the Sun in the past second will be spread out over the surface of a sphere measuring one light-year from center to surface (one light-year in radius). The surface area of a sphere of radius r is $4\pi r^2$. (Instead of a sphere, it may be useful to think of a box centred on the star, with each side of the box $2r$ in length.)

5) Star One has a luminosity L and is located ten light-years away. Star Two also has a luminosity L, but is located twenty light-years away, twice as far away as Star One.

5a) Will Star Two look brighter or fainter than Star One?

5b) How much brighter or fainter will it appear? That is, if we measure a brightness of (say) 1 milliWatt per square kilometer for Star One, what will we measure for Star Two?

So the brightness of a star falls off with increasing distance the same way the brightness of a lamp's light falls off with increasing height above the floor. Both brightnesses fall off with the inverse square of the distance; that is, brightness B is proportional to luminosity L divided by distance^2, or, to be exact, $B = L/4\pi r^2$.

6) Star One and Star Three appear equally bright in the night sky, but *parallax* measurements show that star Three is three times as far away from us as Star One is.

6a) Which star is more luminous, the closer star (One) or the farther star (Three)?

6b) How do you know one star is more luminous?

6c) How much more luminous than the other star is it?