Simulating Martian Gravity

The surface gravity on Mars is smaller than the surface gravity on Earth, primarily because of the smaller mass of Mars as compared to Earth. Objects dropped from above the surface of Mars will thus fall more slowly than they would on Earth. This effect can be simulated on Earth by taking advantage of air resistance. Air resistance can significantly counteract the force of gravity on an object with sufficiently low weight and sufficiently large horizontal surface area. Such objects can fall slowly enough on Earth to mimic objects falling on the surface of Mars. Here we present a simulation and demonstration of that result.

The gravitational acceleration g at the surface of an approximately spherical planet is $g = GM/R^2$, where G is the gravitational constant and M and R are, respectively, the mass and radius of the planet. The importance of both M and R in determining g is illustrated by the fact that Mercury and Mars have the same g. Mercury has both a smaller M and a smaller R than Mars, but in just the right proportion that its g is equal to that of Mars.

The time T required to fall from a height h due to only a gravitational acceleration g can be derived from the equation for distance travelled in time T at a fixed acceleration:

$$h = \frac{1}{2}gT^2 \quad \longrightarrow \quad T(h) = \sqrt{2h/g}. \tag{1}$$

For a non-constant acceleration, as is the case when air resistance significantly opposes gravity, a computer simulation can be used to calculate the the time T.

For Earth, the Moon and Mars (or Mercury), Table 1 gives the gravitational acceleration g as well as the time required to fall from heights of 1, 2 and 3 meters, ignoring air resistance on Earth or on Mars. The MarsSim column gives the fall time results of the simulation developed below.

The net acceleration at time t on a falling object of total mass m_{tot} is a downward (negative) gravitational acceleration plus upward air resistance and buoyancy terms:

$$a(t) = -g + \frac{1}{2}\rho_{\text{air}} C_{\text{drag}} A[v(t)]^2/m_{\text{tot}} + F_{\text{buoy}}/m_{\text{tot}} \tag{2}$$

where the density of air is ρ_{air}, the drag coefficient of the object is C_{drag}, the horizontal cross-sectional area of the object is A, and the object's downward velocity is $v(t)$. The buoyancy force is $F_{\text{buoy}} = m_{\text{air}}g$, where m_{air} is the mass of air displaced by the object.

For our simulation, the object is a hollow rectangular box of vertical depth d, length ℓ and width w (so that $A = \ell w$), with sides of negligible thickness and with mass m_{obj}. For such an object, the mass of air inside the box is $m_{\text{air}} = \rho_{\text{air}}\ell w d$ and the total mass of the falling box is $m_{\rm tot} = m_{\rm obj} + m_{\rm air}$. The drag coefficient for a rectangular box will lie between $C_{\rm drag} = 1.05$ (cube, $\ell = w = d$) and $C_{\rm drag} = 1.17$ (thin rectangular plate with $\ell/w < 5$ and $d \ll \ell$; Hoerner 1965). We assume $C_{\rm drag} = 1.11$ for our rectangular box with $\ell \simeq w$ and $d/\ell \simeq 0.5$. Using the above quantities in the expression for $a(t)$, the velocity $v(t)$ and height fallen $z(t)$ can be found numerically starting from $a(0) = g$, $v(0) = 0$ and $z(0) = h$. The time T required to fall a distance h is found by stopping the calculation when $z(T) = 0$.

For our simulation, we use a large, empty vinyl bag (Richards Homewares Clear Vinyl Jumbo Blanket Bag No. 441W) with $\ell = 0.533\,{\rm m}$, $w = 0.635\,{\rm m}$ and $d = 0.279\,{\rm m}$, so that with $\rho_{\rm air} = 1.225\ {\rm kg\,m^{-3}}$, $m_{\rm air} = 0.116\,{\rm kg}$. The bag itself weighs $m_{\rm obj} = 0.140\,{\rm kg}$. When spray-painted to resemble a Martian rock using a can of orange or reddish paint (e.g., Krylon® Make It Stone!™ Textured Paint, Metallic Copper #48262), the bag weighs $m_{\rm obj} = 0.180\,{\rm kg}$.

Table 1 gives the calculated times for the spray-painted bag to fall distances of 1, 2 and 3 meters on Earth. Those times match the times required for an object to fall from the same heights on Mars to within $\pm 11\%$. The fall time from 2 meters was experimentally verified to be 1.06 ± 0.17 seconds using 3 observers' measurements of 3 separate drops.

This simulation can be used as a straightforward demonstration of how fast objects fall in Martian or Mercurian gravity as compared to Earth gravity by simultaneously dropping, from the same height as the bag, a set of keys or an identical bag filled with textiles.

This simulation can also be used to illustrate the concept of air resistance. Rotating the bag can yield different values of A, resulting in different fall times. Furthermore, fall time measurement distributions, averages and root-mean-square uncertainties can be calculated by having students time one or more drops and record their measurements.

Alternatively, this simulation can be presented as a computational assignment. For example, given an object of fixed volume and mass (matching the spray-painted bag), what horizontal surface areas must it have to fall from 2 meters on Earth and take the same time as an object falling from 2 meters on Mars or the Moon? The simulation for the Mars case could then be performed as a demonstration of the accuracy of the calculation.

REFERENCES

Hoerner, S. F. 1965, Fluid Dynamic Drag, 2nd edition (Bakersfield, CA: Hoerner Fluid Dynamics), 3–16

Table 1: Gravitational Accelerations and Fall Times

Planet	g (m/s^2)	T (1 m)	T (2 m)	T (3 m)
Earth	9.81	0.452	0.639	0.782
Mars	3.70	0.735	1.040	1.274
MarsSim	...	0.656	1.038	1.404
Moon	1.63	1.108	1.567	1.919