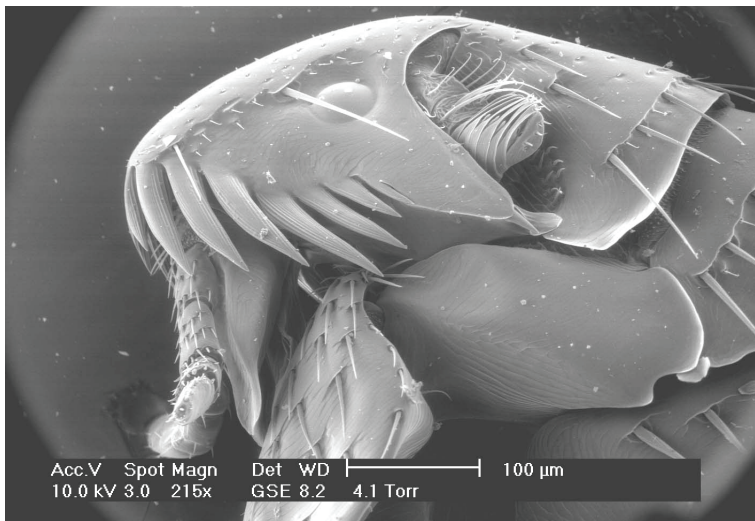


How high can a flea jump? About 20 cm or so, similar to the height that a human can jump. The real question is why do humans and fleas (and other organisms) all jump to approximately the same height. This is what will be explored in this lecture.

Why choose a flea? Well, they are famous jumpers. The typical ‘WOW’ factor is that they jump many times their own height, while we cannot. And, they are deadly as vectors of a number of diseases, of which the Plague is only one example. The larvae feed on detritus that might be found in a house, including the sloughed skin from animals (such as humans). The adults feed mostly on blood. The mouthparts shown below in the scanning electron microphotograph give a sense of how they will chew through what is for them the thick protection of skin, to reach blood vessels for their feast. When their feast is complete, they are engorged, as shown in the CDC microphotograph below.

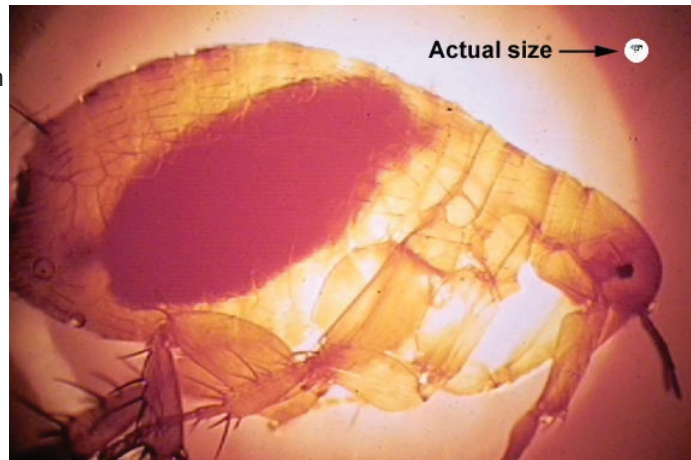


The head of a flea.

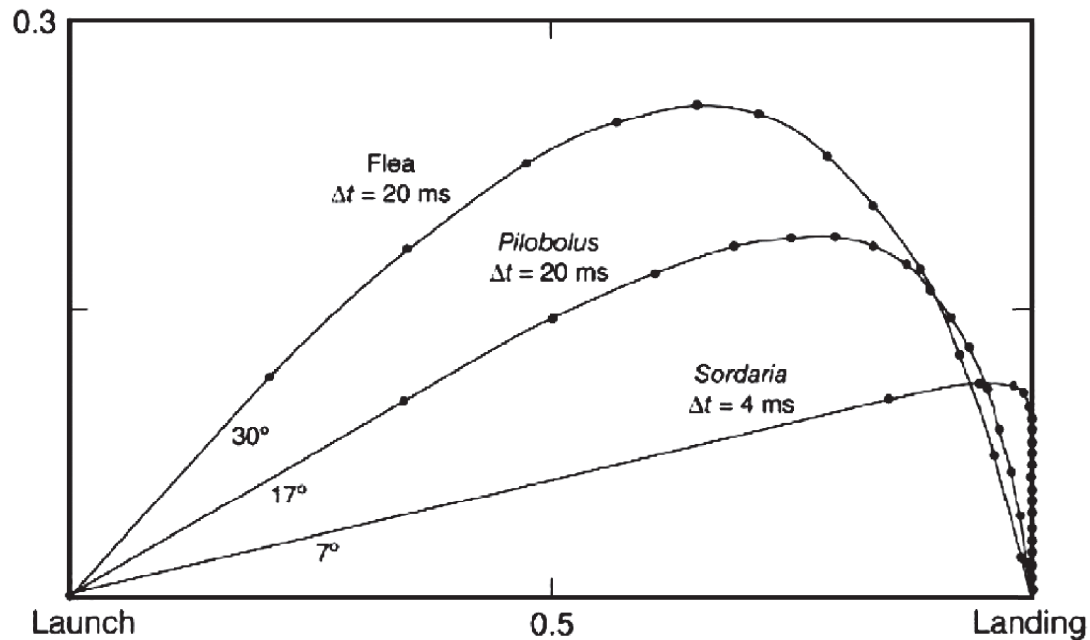
A scanning electron micrograph from The Micro-Environmental Imaging and Analysis Facility at the University of California, Santa Barbara.

Male *Xenopsylla cheopis* (oriental rat flea) engorged with blood. This flea is the primary vector of plague in most large plague epidemics in Asia, Africa, and South America. Both male and female fleas can transmit the infection.

from the CDC Centers for Disease Control and Prevention website. Division of Vector-Borne Infectious Diseases. Plague ("Plague is an infectious disease of animals and humans caused by a bacterium named *Yersinia pestis*.")



Trajectories of a flea jump (as well as trajectories of spores released from fungi) are shown below^[1].

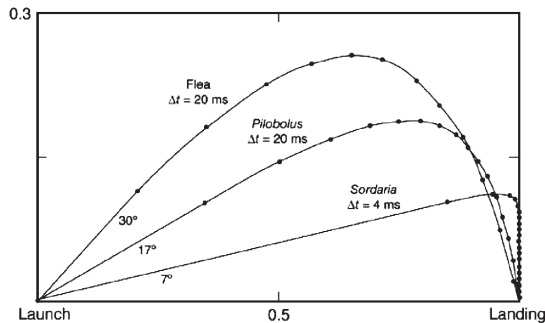


The data have been normalized to maximal distance (the normalized y-scale is expanded 2-fold relative to the x-axis). The absence of a 'well-behaved' parabolic trace (well known in Introductory Physics textbooks) is due to the effect of air resistance to flight.

Bio-ballistic data for various insects and spores. Landing speeds assume launch at the angles that maximize horizontal range and equal launch and landing elevation^[1]. Data for a basketball are shown for comparison.

Projectile	Effective diameter (mm)	Launch speed (m s ⁻¹)	Landing speed (m s ⁻¹)	Launch Reynolds number	Best launch angle	Maximum range (m)	Range loss from drag (%)
Desert locust	10.0	3.0	2.8	2,000	44°	0.85	6.1
Rabbit flea	0.5	4.0	1.0	130	30°	0.3	80.8
Pilobolus	0.3	20.0	1.1	400	17°	0.82	98.0
Sordaria spore	0.04	30.0	0.05	23	7°	0.06	99.96
Basketball	240.5	20.0	18.0	320,000	41.5°	35.7	12.3

^[1]Source: Vogel S (2005) Living in a physical world. II. The bio-ballistics of small projectiles. Journal of Biosciences. 30:167–175



The trajectories don't fit the normal presentation in an Introductory Physics course (where parabolic shapes are usually shown, and the trajectory is calculated as the sum of the vertical and horizontal velocities) because of the dominant effect of air resistance. The initial velocities (launch speeds) must be very high to counteract the effect of air drag^[1].

In a general sense (but with caution, because there are many variations dependent on the organism, and the type of animal), the initial impulse usually results from muscular contractions (fleas use an elastic resilin to store the energy for a leap; mammals can store some of the energy in their tendons). The work exerted will depend upon the speed of the contraction, and the cross-sectional area of the muscle times its length. Muscle contraction speeds are normally in the range of 3 milliseconds. The initial velocity will equal the impulse force divided by the mass ($v = F_{\text{impulse}}/\text{mass}$).

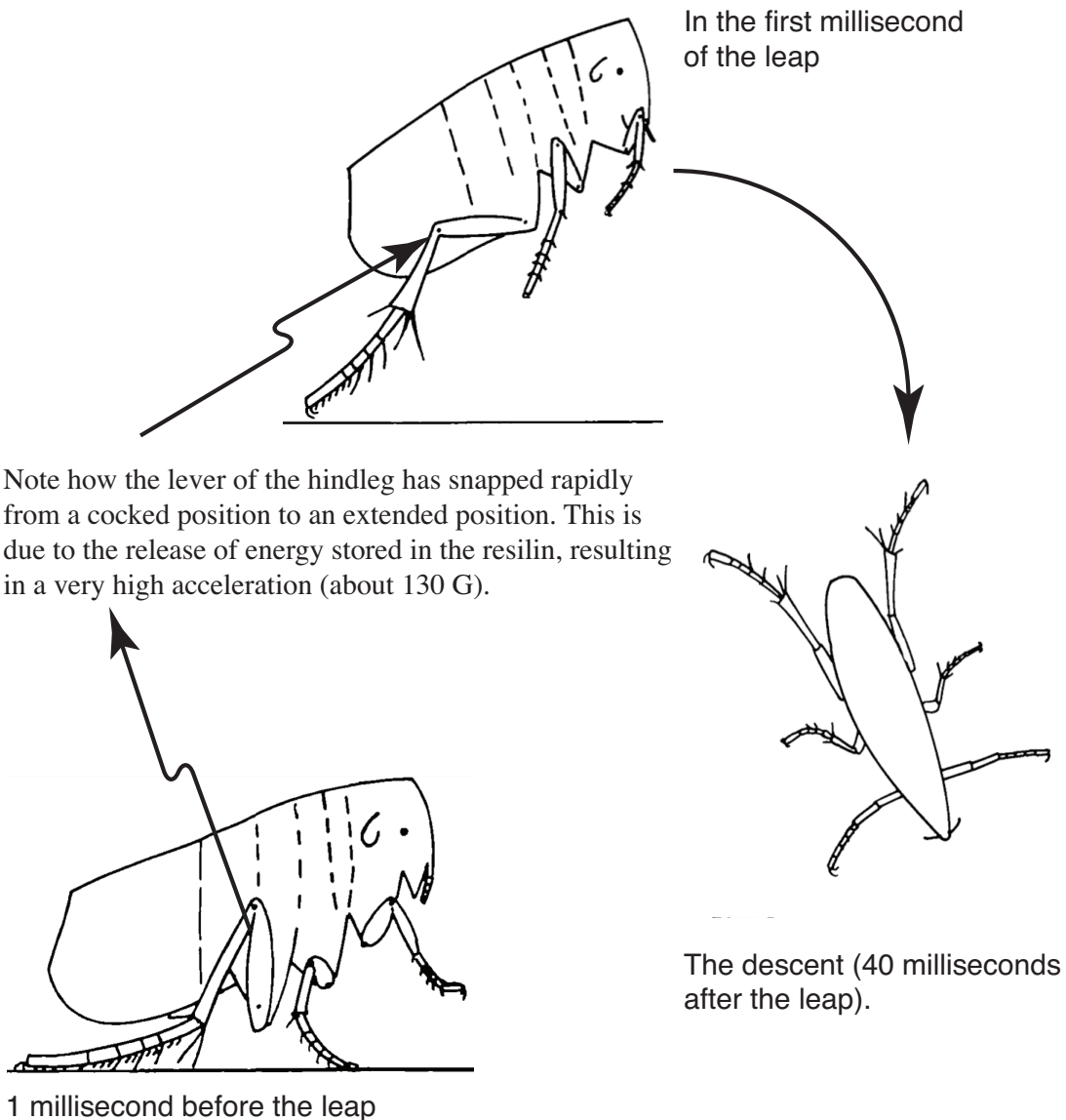
The work done in the leap is proportional to mass and the height of the leap ($W \propto mH$), while the work of the muscles is proportional to the mass of the muscle (or the whole organism) ($W \propto m$). It follows then, that the total work is related solely to the height, since the organism's mass cancels out. Thus, the height of the leap is not proportional to the organisms's size, but rather is similar for any organism. D'Arcy Thompson describes this as an example of the Principle of Biological Similitude.

From a biological perspective, it's the exceptions that evoke fascination. For example, a flea jumps only 20 cm (or so), but a leopard can leap about 2 meters. Why? Air resistance is part of the answer, as it relates to limitations to the F_{impulse} . The acceleration of the flea to reach an initial velocity of 100 cm sec^{-1} is about $1.33 \cdot 10^5 \text{ cm} \cdot \text{sec}^{-2}$ (the mass of the flea is about 0.45 mg)^[2]. For man (or a leopard), the F_{impulse} is far lower, about $1.5 \cdot 10^2 \text{ cm} \cdot \text{sec}^{-2}$. For a flea, it's about 135 G; for man or leopard, about 1.5 G. Such a high G-force causes a tremendous strain on the structural elements of the flea, and may explain why its leaping height is limited.

^[1]Source: Vogel S (2005) Living in a physical world. II. The bio-ballistics of small projectiles. Journal of Biosciences. 30:167–175

^[2]Source: Bennet-Clark HC and ECA Lucey (1967) The jump of a flea: A study of the energetics and a model of the mechanism. Journal of Experimental Biology 47:59–76.

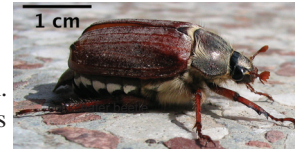
As noted before, leaping involves a diversity of variations dependent on the organism, and the type of animal). The flea stores energy in an elastic material (resilin) to allow it to generate such a high F_{impulse} . Shown below are snapshots of the leap of a flea taken with a high speed camera running at up to 8000 frames per second^[1].



^[1]Source: Bennet-Clark HC and ECA Lucey (1967) The jump of a flea: A study of the energetics and a model of the mechanism. *Journal of Experimental Biology* 47:59–76.

Size and Jumping

“Leaving aside the question of the supply of energy, and keeping to that of the mechanical efficiency of the machine, we may find endless biological illustrations of the principle of similitude. All through the physiology of locomotion we meet with it in various ways: as, for instance, when we see a cockchafer carry a plate many times its own weight upon its back, or a flea jump many inches high. 'A dog,' says Galileo, 'could probably carry two or three such dogs upon its back; but I believe that a horse could not carry even one of his own size.'”



Such problems were admirably treated by Galileo and Borelli, but many writers remained ignorant of their work. Linnaeus remarked that if an elephant were as strong in proportion as a stagbeetle, it would be able to pull up rocks and level mountains; and Kirby and Spence have a well-known passage directed to show that such powers as have been conferred upon the insect have been withheld from the higher animals, for the reason that had these latter been endowed therewith they would have 'caused the early desolation of the world'^[1].

Such problems as that presented by the flea's jumping powers^[2], though essentially physiological in their nature, have their interest for us here: because a steady, progressive diminution of activity with increasing size would tend to set limits to the possible growth in magnitude of an animal just as surely as those factors which tend to break and crush the living fabric under its own weight. In the case of a leap, we have to do rather with a sudden impulse than with a continued strain, and this impulse should be measured in terms of the velocity imparted. The velocity is proportional to the impulse (x), and inversely proportional to the mass (M) moved: $V = x/M$. But, according to what we still speak of as 'Borelli's law', the impulse (i.e. the work of the impulse) is proportional to the volume of the muscle by which it is produced,^[3] that is to say (in similarly constructed animals) to the mass of the whole body; for the impulse is proportional on the one hand to the cross-section of the muscle, and on the other to the distance through which it contracts. It follows from this that the velocity is constant, whatever the size of the animal.

Putting it still more simply, the work done in leaping is proportional to the mass and to the height to which it is raised, $W \propto mH$. But the muscular power available for this work is proportional to the mass of muscle, or (in similarly constructed animals) to the mass of the animal, $W \propto m$. It follows that H is, or tends to be, a constant. In other words, all animals, provided always that they are similarly fashioned, with their various levers in like proportion, ought to jump not to the same relative but to the same *actual* height.^[4] The grasshopper seems as well planned for jumping as the flea, and the flea's jump is about 200 times its own height, but the grasshopper's at most 20-30 times; and neither flea nor grasshopper is a better but rather a worse jumper than a horse or a man^[1].”

D'Arcy Wentworth Thompson (1961) *On Growth and Form*. (ed. John Tyler Bonner). Cambridge University Press. pp. 26-28.

^[1] *Introduction to Entomology*, II (1826), 190. Kirby and Spence, like many less learned authors, are fond of popular illustrations of the 'wonders of Nature', to the neglect of dynamical principles. They suggest that if a white ant were as big as a man, its tunnels would be 'magnificent cylinders of more than three hundred feet in diameter'; and that if a certain noisy Brazilian insect were as big as a man, its voice would be heard all the world over, 'so that Stentor becomes a mute when compared with these insects!'. It is an easy consequence of anthropomorphism, and hence a common characteristic of fairy tales, to neglect the dynamical and dwell on the geometrical aspect of similarity.

^[2] The flea is a very clever jumper; he jumps backwards, is streamlined accordingly, and alights on his two hind-legs.

^[3] That is to say, the available energy of muscle, in ft-lb per lb of muscle, is the same for all animals: a postulate which requires considerable qualification when we come to compare very different kinds of muscles, such as the insect's and the mammal's.

^[4] Borelli, *Prop. CLXXVII*. *Animalia minora et minus ponderosa majores saltus efficiunt respectu sui corporis, si caetera fuerint paria*.

^[1] The high-jump is nowadays a highly skilled performance. For the jumper contrives that his centre of gravity goes under the bar, while his body, bit by bit, goes over it.