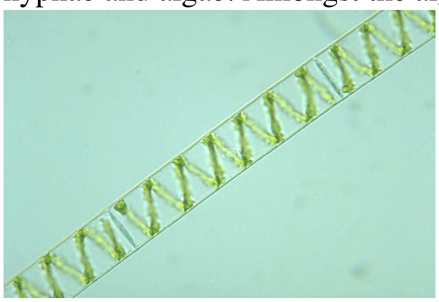
SC/BIOL 2090.02 - Current Topics in Biophysics

ASSIGNMENT ONE

There are two questions. You must complete both.

QUESTION ONE

As presented in lecture, the ratio of surface area to volume is a key element of biological shape and form, since some physiological functions scale with surface area while others scale with volume. The surface area to volume ratios of cubes and spheres are simple to evaluate, but rectangular shapes pose problems, as do the ratios for a cylinder (if the surface area of the circular top and bottom are included). Many organisms are cylindrical in shape; for example, the filamentous forms of fungal hyphae and algae. Amongst the algae, the size of the



cylindrical form varies a lot: from exceedingly long narrow cylindrical filaments about 20 μm in diameter (*Spirogyra* spp., left) to the relatively wide cylinders (about 5000 μm in diameter) for the internodal cells of *Chara* spp. (right). Evaluate the surface area to volume ratio of a cylindrical form that includes the areas at the top and bottom of the cylinder for various ratios of the radius to height.

Hints: The surface area of a cylinder is equal to $2 \bullet \pi \bullet r^2 + 2 \bullet \pi \bullet r \bullet h$ (where r is the radius and h is the height), and the volume is equal to $\pi \bullet r^2 \bullet h$. A graphic plot is likely to be crucial (A/V *versus* h/r is one possibility) with diagrams of the cylindrical shapes.

QUESTION TWO

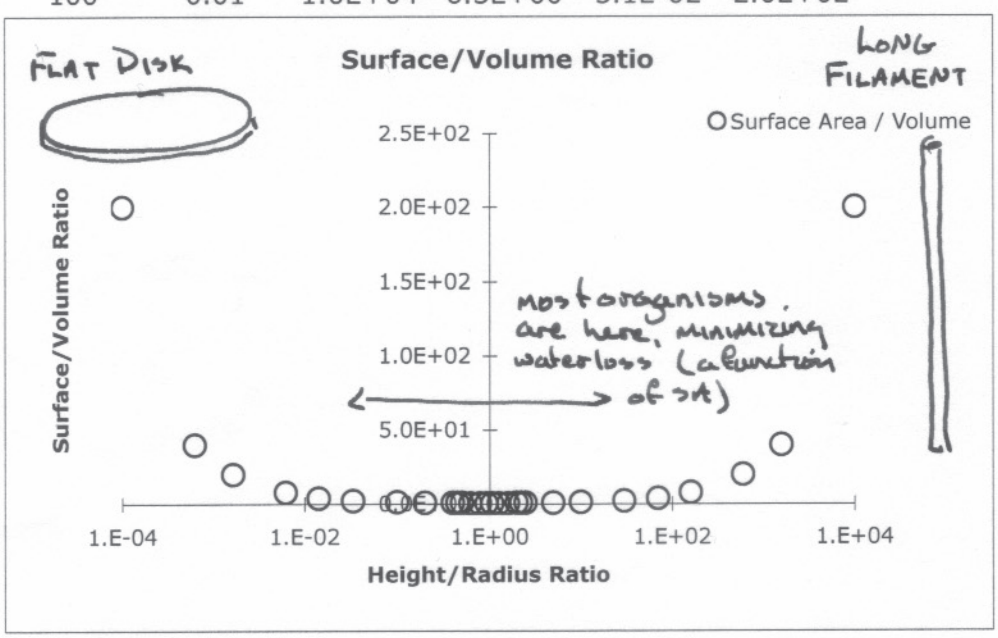
For the case of a Dyson tree (see course notes), propose a mechanism for moving water from the comet surface to the photosynthetic apparatus at the 'top' of the tree (and back again, since this is a closed system). Your mechanism must rely on already existent anatomical features. It must be physically realistic (you are required to show that it is).

Hint: I wonder if a heat gradient (evaporative) might work, resulting in a pressure gradient sufficient to move water through xylem for a tree as tall as a Dyson tree.

Guidelines: I expect that students may wish to work together on the assignment, that is fine, but, be sure that your assignment is in your own words. Remember that you have to explain your answers with sufficient clarity, so that a non-physicist like Dr. Lew will understand them. He often finds diagrams helpful and is obsessed with ensuring that the units work, so showing the units is obligatory. Excessive length is not encouraged.

Question One

height h	radius r	h/r	SA	volume	SA/vol
0.01	100	1.0E-04	6.3E+04	3.1E+02	2.0E+02
0.05	80	6.3E-04	4.0E+04	1.0E+03	4.0E+01
0.1	60	1.7E-03	2.3E+04	1.1E+03	2.0E+01
0.25	40	6.3E-03	1.0E+04	1.3E+03	8.1E+00
0.5	35	1.4E-02	7.8E+03	1.9E+03	4.1E+00
1	30	3.3E-02	5.8E+03	2.8E+03	2.1E+00
2	20	1.0E-01	2.8E+03	2.5E+03	1.1E+00
3	15	2.0E-01	1.7E+03	2.1E+03	8.0E-01
4	10	4.0E-01	8.8E+02	1.3E+03	7.0E-01
4.25	9.5	4.5E-01	8.2E+02	1.2E+03	6.8E-01
4.5	9	5.0E-01	7.6E+02	1.1E+03	6.7E-01
5	8	6.3E-01	6.5E+02	1.0E+03	6.5E-01
5.5	7	7.9E-01	5.5E+02	8.5E+02	6.5E-01
6	6.5	9.2E-01	5.1E+02	8.0E+02	6.4E-01
6.5	6	1.1E+00	4.7E+02	7.3E+02	6.4E-01
7	5.5	1.3E+00	4.3E+02	6.6E+02	6.5E-01
8	5	1.6E+00	4.1E+02	6.3E+02	6.5E-01
9	4.5	2.0E+00	3.8E+02	5.7E+02	6.7E-01
9.5	4.25	2.2E+00	3.7E+02	5.4E+02	6.8E-01
10	4	2.5E+00	3.5E+02	5.0E+02	7.0E-01
15	3	5.0E+00	3.4E+02	4.2E+02	8.0E-01
20	2	1.0E+01	2.8E+02	2.5E+02	1.1E+00
30	1	3.0E+01	1.9E+02	9.4E+01	2.1E+00
35	0.5	7.0E+01	1.1E+02	2.7E+01	4.1E+00
40	0.25	1.6E+02	6.3E+01	7.9E+00	8.1E+00
60	0.1	6.0E+02	3.8E+01	1.9E+00	2.0E+01
80	0.05	1.6E+03	2.5E+01	6.3E-01	4.0E+01
100	0.01	1.0E+04	6.3E+00	3.1E-02	2.0E+02



((2*3.14*r^2)+(2*3.14*r*h))/(3.14*r^2*h)

QUESTION TWO

For the case of a Dyson tree (see course notes --below right), propose a mechanism for moving water from the comet surface to the photosynthetic apparatus at the 'top' of the tree (and back again, since this is a closed system). Your mechanism must rely on already existent anatomical features. It must be physically realistic (you are required to show that it is).

Hint: I wonder if a heat gradient (evaporative) might work, resulting in a pressure gradient sufficient to move water through xylem for a tree as tall as a Dyson tree.

Rubric: Examples of biophysical tests of the various aspects of the movement of water to a top of a tree are shown on the next page. The most correct answer is that evaporative pull can work, but the tensile strength of water limits the tree height to far less than 'hundreds of miles'.

However, the range of answers from students was far broader. This was not surprising given the open-ended nature of the problem.

Grading of their assignments required an <u>adaptive rubric</u> that accounted for the approaches that they choose to use. Besides clarity and logic, the approaches had to be anchored in biological and biophysical reality.

The physicist and futurist, Freeman Dyson, proposed the possibility of extraterrestrial colonization of comets ("...it is likely that space around the solar system is populated by huge numbers of comets, small worlds a few miles in diameter, rich in water and the other chemicals essential to life."[1]) by using genetically engineered trees with modifications suitable for survival in the rigour –cold and vacuum– of space (Dyson trees[2]). Thus, the leaves would be impermeant to gases but transparent to >400 nm light, and opaque to damaging ultraviolet. Water and CO_2 required for photosynthesis would be extracted from the comet itself, while the O_2 produced as a byproduct of photosynthesis would provide for heterotrophic life forms; for example, humans. The idea of trees growing on comets exists in other forms, such as the Baobab trees out-growing their very small planets in the children's story, The Little Prince[3].

Dyson considered the height of a tree[1]: "How high can a tree on a comet grow? The answer is surprising. On any celestial body whose diameter is of the order of ten miles or less, the force of gravity is so weak that a tree can grow infinitely high. Ordinary wood is strong enough to lift its own weight to an arbitrary distance from the center of gravity. This means that from a comet of ten-mile diameter, trees can grow out for hundreds of miles, collecting the energy of sunlight from an area thousands of times as large as the area of the comet itself. Seen from far away, the comet will look like a small potato sprouting an immense growth of stems and foliage. When man comes to live on the comets, he will find himself returning to the arboreal existence of his ancestors."

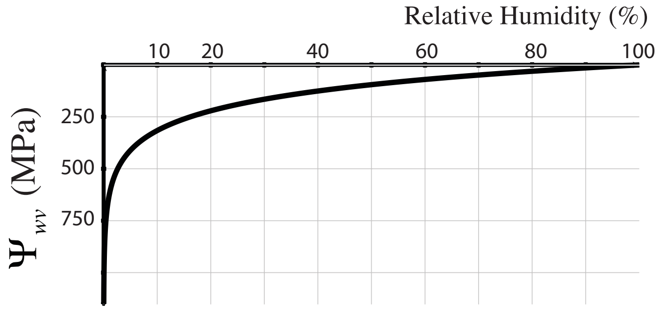
Work Problem:
Is Freeman Dyson right about the height of Dyson trees?

[1]From Dyson Freeman's essay "The World, The Flesh, and the Devil" republished in The Scientist as Rebel (2006) New York Review Books.
[2]http://en.wikipedia.org/wiki/Dyson_tree
[3]The Baobab. From The Little Prince by Antoine de Saint-Exupery (1943)

Biophysical calculations

1) Evaporation

$$\Psi_{wv} = \frac{RT}{\overline{V}_w} \ln\left(\frac{\%\ \text{relative humidity}}{100}\right) + \rho_w \cdot g_{comet} \cdot h$$



$\frac{RT}{\overline{V}_w}$ is 137.3 MPa at room temperature (25 degrees Celsius)

At a low enough relative humidity (2%) $\Psi_{\text{water vapour}}$ is about -500 MPa.

2) Water flow through xylem as long as a Dyson tree (say, 100 km)

For a tube diameter of about 200 μm and a flow rate of about 100 meter per hour, the required pressure gradient is 0.01 MPa per meter.

(0.01 MPa m^{-1})(10^5 m) = 1000 MPa!

but note that decreasing the flow rate to 50 meters per hour means that the suction to pull ($\Psi_{\text{water vapour}}$ is about -500 MPa) and the pressure required for the flow rate (about -500 MPa) are similar.

3) Is water strong enough?

If the tensile strength is 30–50 MPa, then no, it is not.
We could scale back the height of the tree. About 10 km would be in the right range.

4) Capillary rise?

To explore this we equate the capillary tension with the 'weight' of the water column.

surface tension *gravitational*

$$\gamma(2 \cdot \pi \cdot r) = \rho \cdot h(\pi \cdot r^2) g_{comet}$$

$$h = \frac{2 \cdot \gamma}{\rho \cdot r \cdot g_{comet}}$$

surface tension of water
~70 dyne/cm (7.0×10^{-2} N m^{-1}).
N=kg m s^{-2}, so 7.0×10^{-2} kg s^{-2}

$$g_{comet} = \frac{G \cdot M}{r^2} = \frac{(6.67 \times 10^{-11}\ \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{sec}^{-2})(\frac{4}{3}\pi r^3 \cdot 1\text{kg} \cdot \text{m}^{-3})}{r^2}$$

For a comet of radius 5000 m, $g_{comet} = 1.39 \times 10^{-6}\ \text{m} \cdot \text{sec}^{-2}$

The height (h) is 10^7 km. **But, would it flow fast enough?** Using surface tension (7.0×10^{-2} N m^{-1}) to calculate the pressure gradient of the 100 km tree:

$$\Delta P = (7.0 \times 10^{-2}\ \text{N m}^{-1})/(10^4\ \text{m}) = 7.0 \times 10^{-6}\ \text{Pa}).$$

That is, very low, and thus very slow. In addition capillary rise does not account for the need to remove water at the top of the Dyson tree.