ASSIGNMENT ONE

(due Thursday 26 September at Farquharson 229 before 5:00 PM)

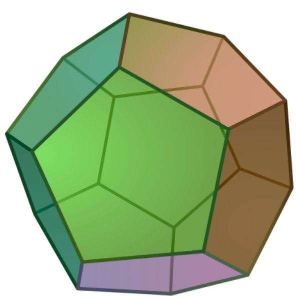
Question One. *Radiative Balance*

The power output of the sun is approximately 27 Watts m^{-3}, while an adult human has a power output of approximately 800 Watts m^{-3}. Yet, the temperature of the sun is approximately 5778 K (at the surface) while human temperature is about 37 ºC (310 K). How come? Explain why so that even a non-physicist like Dr. Lew can understand the mathematical analysis. Please ensure you show units!

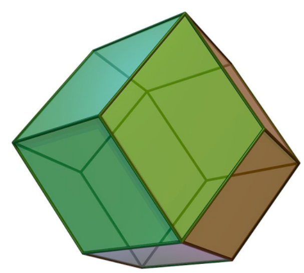
Hints and facts that you may (or may not) find useful...

- You may wish to take a look at your course notes related to one aspect of the problem (radiative balance)
- The volume of the sun is 1.412×10^{18} km^3. Its average density is 1408 kg/m^3. Its spherical diameter is 1.392684×10^{6} km.
- The average density of an adult human is 1062 kg/m^3. Its average weight (globally, there are significant regional variations) is 62 kg. The average height of a Canadian human is 1.751 m (male) 1.623 m (female). Wikipedia provides an article on estimation and measurement of surface area of humans — (en.wikipedia.org/wiki/Body_surface_area).

Question Two. *Area to Volume Ratios*



For a dodecahedron (left) and a rhombic dodecahedron (right), graph the area *versus* volume on a log-log scale and determine the slope. Compare to a cube and a sphere. Explain why the rhombic dodecahedron may have biological relevance (provide a biological example, if possible).



Hints and facts that you may (or may not) find useful...

- You should be able to find relevant information in sources like Wikipedia. D'Arcy Thompson (*On Growth and Form*) discusses biological relevance. I'm not sure if this is easily searchable on the internet.

Guidelines

I expect that students may wish to work together on the assignment, that is fine, but be sure that your assignment is in your own words. Remember that you have to explain your answers with sufficient clarity, so that a non-physicist like Dr. Lew will understand them. He often finds diagrams helpful and is obsessed with ensuring that the units work, so showing the units is obligatory. Excessive length is not encouraged.

Assignment Key (Rubric Overleaf)

Even though humans have denser energy production than the sun, the sun is a lot larger with a much smaller surface area / volume ration.

Besides exploring the radiant balance, here are some aspects of the problem. The energy density ratio

$$\frac{\text{sun}}{\text{human}} \quad \frac{27\ W\,m^{-3}}{800\ W\,m^{-3}} = 0.0338$$

But, the sun is hotter: $\frac{\text{sun}}{\text{human}} \quad \frac{5778}{310} = 18.6$

Neither one of these ratios is very large, but the $\frac{\text{surface area}}{\text{volume}}$:

Sun $\quad \frac{\text{surface area}}{\text{volume}} \quad \frac{4\pi r^2}{1.412 \times 10^{18}\ km^3} \longrightarrow 4\pi(0.7 \times 10^6)^2$

$$\longrightarrow \frac{6.15 \times 10^{12}\ km^2}{1.412 \times 10^{18}\ km^3} = 4.36 \times 10^{-6}\ km^{-1} \qquad \frac{1\ km}{10^3\ m}$$

$$(4.36 \times 10^{-8}\ m^{-1})$$

Human $\quad \frac{\text{surface area}}{\text{volume}} = \frac{1.75\ m^2}{0.058\ m^3} = 30.17\ m^{-1}$ (from wikipedia) $\updownarrow$ 7×10^8-fold

is very different (7×10^9-fold difference)

Radiant output (per m^2) is much denser for the sun, astronomically so (pun intended)!

Does the sun comply with Kleiber's rule $\frac{\text{metabolism}}{\text{kg}} \propto \text{kg}^{-3/4}$

For a human $\frac{800\ W}{m^3} = \alpha\,(0.05)^{-3/4}$

$\quad \hookrightarrow \alpha = 84.5$

For the sun $\frac{W}{m^3} \approx 84.5\,(1.42 \cdot 10^{27})^{-3/4}$

$\approx 3.7 \times 10^{-19}\ W/m^3$

Therefore, The Sun is Not a Mammal!

Rubric

One

1	$P = \sigma \epsilon A (T_{body}^4 - T_{sun}^4)$	30
2	Total watts $W/m^3 \cdot m^3$	30
3	$\frac{\text{surface area}}{\text{volume}}$ ratio	30
4.	Additional	10
		100

Two

1	$2/3$ (or $3/2$) slope	50
2	tesselation (packing)	50
		100