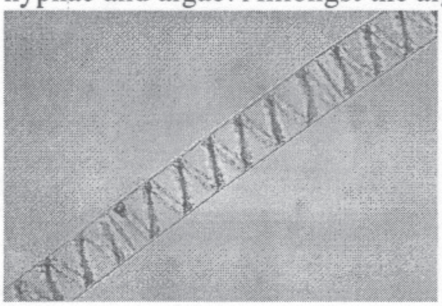
SC/BIOL 2090.02 - Current Topics in Biophysics

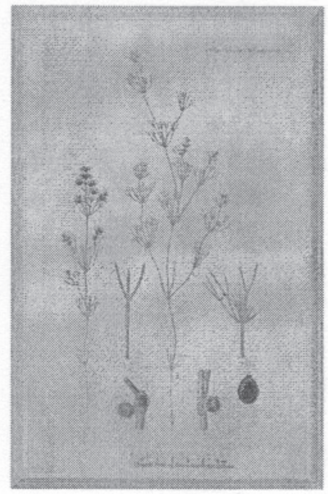
ASSIGNMENT ONE

There are two questions. You must complete both.

QUESTION ONE

As presented in lecture, the ratio of surface area to volume is a key element of biological shape and form, since some physiological functions scale with surface area while others scale with volume. The surface area to volume ratios of cubes and spheres are simple to evaluate, but rectangular shapes pose problems, as do the ratios for a cylinder (if the surface area of the circular top and bottom are included). Many organisms are cylindrical in shape; for example, the filamentous forms of fungal hyphae and algae. Amongst the algae, the size of the cylindrical form varies a lot: from exceedingly long narrow cylindrical filaments about 20 μm in diameter (*Spirogyra* spp., left) to the relatively wide cylinders (about 5000 μm in diameter) for the internodal cells of *Chara* spp. (right). Evaluate the surface area to volume ratio of a cylindrical form that includes the areas at the top and bottom of the cylinder for various ratios of the radius to height.





Hints: The surface area of a cylinder is equal to $2 \cdot \pi \cdot r^2 + 2 \cdot \pi \cdot r \cdot h$ (where r is the radius and h is the height), and the volume is equal to $\pi \cdot r^2 \cdot h$. A graphic plot is likely to be crucial (A/V *versus* h/r is one possibility) with diagrams of the cylindrical shapes.

QUESTION TWO

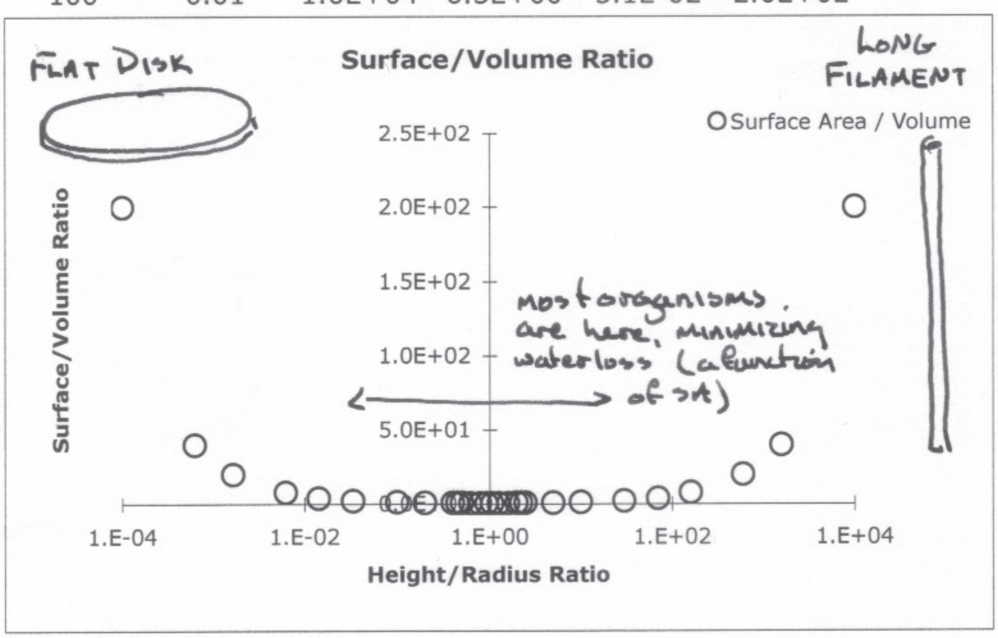
For the case of a Dyson tree (see course notes), propose a mechanism for moving water from the comet surface to the photosynthetic apparatus at the 'top' of the tree (and back again, since this is a closed system). Your mechanism must rely on already existent anatomical features. It must be physically realistic (you are required to show that it is).

Hint: I wonder if a heat gradient (evaporative) might work, resulting in a pressure gradient sufficient to move water through xylem for a tree as tall as a Dyson tree.

Guidelines: I expect that students may wish to work together on the assignment, that is fine, but, be sure that your assignment is in your own words. Remember that you have to explain your answers with sufficient clarity, so that a non-physicist like Dr. Lew will understand them. He often finds diagrams helpful and is obsessed with ensuring that the units work, so showing the units is obligatory. Excessive length is not encouraged.

Question One

height h	radius r	h/r	SA	volume	SA/vol
0.01	100	1.0E-04	6.3E+04	3.1E+02	2.0E+02
0.05	80	6.3E-04	4.0E+04	1.0E+03	4.0E+01
0.1	60	1.7E-03	2.3E+04	1.1E+03	2.0E+01
0.25	40	6.3E-03	1.0E+04	1.3E+03	8.1E+00
0.5	35	1.4E-02	7.8E+03	1.9E+03	4.1E+00
1	30	3.3E-02	5.8E+03	2.8E+03	2.1E+00
2	20	1.0E-01	2.8E+03	2.5E+03	1.1E+00
3	15	2.0E-01	1.7E+03	2.1E+03	8.0E-01
4	10	4.0E-01	8.8E+02	1.3E+03	7.0E-01
4.25	9.5	4.5E-01	8.2E+02	1.2E+03	6.8E-01
4.5	9	5.0E-01	7.6E+02	1.1E+03	6.7E-01
5	8	6.3E-01	6.5E+02	1.0E+03	6.5E-01
5.5	7	7.9E-01	5.5E+02	8.5E+02	6.5E-01
6	6.5	9.2E-01	5.1E+02	8.0E+02	6.4E-01
6.5	6	1.1E+00	4.7E+02	7.3E+02	6.4E-01
7	5.5	1.3E+00	4.3E+02	6.6E+02	6.5E-01
8	5	1.6E+00	4.1E+02	6.3E+02	6.5E-01
9	4.5	2.0E+00	3.8E+02	5.7E+02	6.7E-01
9.5	4.25	2.2E+00	3.7E+02	5.4E+02	6.8E-01
10	4	2.5E+00	3.5E+02	5.0E+02	7.0E-01
15	3	5.0E+00	3.4E+02	4.2E+02	8.0E-01
20	2	1.0E+01	2.8E+02	2.5E+02	1.1E+00
30	1	3.0E+01	1.9E+02	9.4E+01	2.1E+00
35	0.5	7.0E+01	1.1E+02	2.7E+01	4.1E+00
40	0.25	1.6E+02	6.3E+01	7.9E+00	8.1E+00
60	0.1	6.0E+02	3.8E+01	1.9E+00	2.0E+01
80	0.05	1.6E+03	2.5E+01	6.3E-01	4.0E+01
100	0.01	1.0E+04	6.3E+00	3.1E-02	2.0E+02

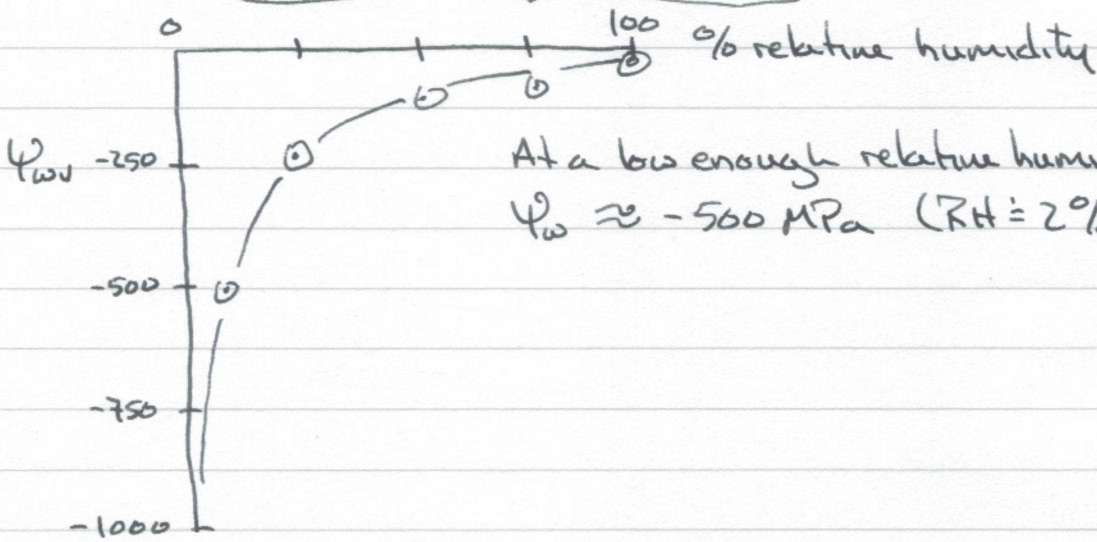


((2*3.14*r^2)+(2*3.14*r*h))/(3.14*r^2*h)

Can water be pulled through a Dyson tree?

1) EVAPORATION

$$\Psi_w = \frac{RT}{\bar{V}_w} \ln\left[\frac{\%\text{ relative humidity}}{100}\right] + \rho_w \cdot g_{comet} \cdot h$$



At a low enough relative humidity, $\Psi_w \approx -500$ MPa (RH ≐ 2%)

2) WATER FLOW THROUGH XYLEM AS LONG AS A DYSON TREE (say, 100 km).

For a tube diameter of about 200 μm and a flow rate of about 100 meters/hour,

$$\frac{\Delta P}{l} = \frac{0.01\text{ MPa}}{m} \quad (0.01\text{ MPa m}^{-1}) \cdot (10^5\text{ m})$$

is 1000 MPa !

but note that decreasing the flow rate to 50 meters/hr means that the suction to pull & required flow rates (~500 MPa) would be similar.

3) IS WATER STRONG ENOUGH?

If the tensile strength is 30-50 MPa
Then no, it is not.
We could scale back the height of the tree.
About 10 km would be in the right range.

4) CAPILLARY RISE?

$$\underbrace{\gamma(2\pi r)}_{\text{surface tension}} = \underbrace{\rho \cdot h (\pi r^2) g_{comet}}_{\text{gravitational}}$$

$$h = \frac{2\gamma}{\rho \cdot r \cdot g_{comet}}$$

surface tension of water: ~70 dyne/cm
or 7.0×10^{-2} N/m $N = kg \cdot m \cdot sec^{-2}$
so $7.0 \times 10^{-2}\ kg \cdot sec^{-2}$

comet gravity

$$g_{comet} = \frac{G \cdot M}{r^2}$$

gravitational constant $6.67 \times 10^{-11}\ m^3/kg \cdot sec^2$

comet mass $(\tfrac{4}{3}\pi r^3)(1\ kg \cdot m^{-3})$

r = 5000 m

$M = 5.23 \times 10^{11}$ kg.

$g_{comet} = 1.39 \times 10^{-6}\ m\ sec^{-2}$

$$\text{height, } h = \frac{2 \cdot 7 \times 10^{-2}\ kg \cdot sec^{-2}}{(1\ kg\,m^{-3})(10^{-4}\ m)(1.39 \times 10^{-6}\ m \cdot sec^{-2})} = 10^{9} \text{ meters}$$

BUT, how long would it take?

4) Surface Tension $\longleftrightarrow$ Pressure.

$$N = Pa \cdot m^2.$$

$$\gamma = 7.0 \times 10^{-2} \; N/m$$

$$= 7.0 \times 10^{-2} \; Pa \cdot m^2/m \quad \text{or} \quad Pa \cdot m$$

For a 10 km tree.

$$\Delta P = \frac{7.0 \times 10^{-2} \; P \cdot m}{10^4 \; m} = 7 \times 10^{-6} \; Pa.$$

- very low
- very slow.