ACCEPTANCE SAMPLING

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Introduction

Sampling is widely used in industry and government for controlling the quality of shipments of components, supplies, raw materials, and final products.

Consider a company receiving shipments of parts. These parts will eventually be assembled with other purchased parts or products of own manufacture. Some of the parts will be defective; if not detected earlier, these defective parts will cause the product into which they are assembled also to be defective. The company recognizes that it is difficult to insist on zero defectives and is willing to accept a lot provided that it does not contain too many defectives.

In general, the number marking the boundary between "a few" and "too many" defectives, (the maximum acceptable number of defectives) varies depending on the situation. This number depends on the lot size, the cost of inspecting and testing a part and an assembly, the cost of dismantling and repairing an assembly, the loss associated with the possible failure to meet customer requirements, etc. In the pharmaceutical industry, for example, where a "defective"—say, a spoiled tetanus vaccine—may mean the death or injury of a patient, defectives are not tolerated at all, and stringent tests are made at every step to screen them out. In other industries, however, where the consequence of an undetected defective item may not be so serious, firms, in the interest of speed and convenience, usually accept lots even when some of the items in the lot are likely to be defective.

In order to control the quality of purchased lots, two major alternatives are open to a buyer. One, complete inspection: every single item in the lot is inspected and tested. Two, partial inspection: a sample of items is taken, the sampled items are inspected and tested, and the lot as a whole is accepted or rejected depending on whether few or many defective items are found in the sample. This type of sampling—one of many used to control the quality of manufacturing processes or lots—is known as *acceptance sampling.*

Complete inspection is often impractical, uneconomical, or impossible—for a number of reasons. The test itself may destroy the item being tested (for example, when light bulbs are tested for life duration, the test consists of letting the bulb burn until it burns out), the inspection costs may be too high, there may not be enough trained inspectors, inspection may be too time consuming, and so on. Unless the identification of every single defective item is considered essential, a sample may be preferable to complete inspection.

Of course, when a decision about the lot is based on a sample, certain risks are undertaken. A lot containing a small number of defective items may be rejected because too many of these defectives happened to show up in the sample, or a lot containing a large number of defectives may be accepted because too few defectives happened to show up in the sample. These risks are inherent in the sampling procedure itself and cannot be entirely eliminated.

When a lot is accepted, the items become part of the buyer's stocks and remain in inventory until they are sold, assembled, processed or otherwise disposed of. When a lot is rejected, however, its fate depends on whatever agreements were made with the seller. For example, the lot may be returned to the seller, who then inspects or tests some or all of the items and replaces any defective items found; the partially or totally corrected lot is then returned to the buyer, who may inspect the new lot completely or form a decision on the basis of the results from another sample taken from the lot. Alternatively, the buyer may inspect all items left in the lot that were not part of the sample, and return to the seller for replacement all defectives found in the original sample and in the rest of the lot. Numerous other possibilities exist.

Sampling plans

It is clear from all this that a buyer rarely can act unilaterally and implement a control procedure without the consent of the seller. In the majority of cases, both parties must reach an agreement on the acceptable quality level, the manner in which inspection is carried out, the sampling plan to be used, and their mutual obligations.

One of the sampling schemes most widely used in industry and government was initially prepared by a working group representing the military services of Canada, the United Kingdom, and the United States, and is best known under its U.S. title: *Military Standard 105E.** For convenience, we shall refer to it as "the standard" throughout this reading.

The standard was originally intended to facilitate negotiations between a supplier and a military agency. It has become, however, quite popular in dealings among non-military government agencies, private sellers and buyers. The use of the standard is stipulated in many government procurement contracts in the U.S. and other countries. The sampling plans contained in the standard are arranged very conveniently and are very easy to select and apply. The standard's ease of use and its widespread acceptance among manufacturers, in turn, account for its use in contracts between private buyers and sellers.

In the standard, three general levels of inspection are distinguished. Level II is designated as "normal". Level I is recommended when "less discrimination" is needed, as in testing lots of relatively inexpensive items, while level III is recommended when "more discrimination" is needed and may be suitable for inspection of relatively expensive and important items.

In early negotiations between a buyer and a seller it is expected that an agreement will be reached as to which level of inspection is appropriate. An agreement

* Related to this standard are ANSI/ASQC Z1.4 and ISO 2859.

must also be reached concerning the *acceptable quality level (AQL)* of the product to be supplied. This is the maximum percentage of defective items in the lot that the buyer will tolerate. Given the size of the lot, and an agreed upon AQL and inspection level, the standard provides a set of tables from which the sampling plan can be obtained. Table 1 and Figure 1 are extracted from the standard.

Table 1
Sample size codes

Lot or batch size			Inspection levels		
			I	II	III
2	to	8	A	A	B
9	to	15	A	B	C
16	to	25	B	C	D
26	to	50	C	D	E
51	to	90	C	E	F
91	to	150	D	F	G
151	to	280	E	G	H
281	to	500	F	H	J
501	to	1200	G	J	K
1201	to	3200	H	K	L
3201	to	10000	J	L	M
10001	to	35000	K	M	N
35001	to	150000	L	N	P
150001	to	500000	M	P	Q
500001	and	over	N	Q	R

Table 1 supplies a code letter for the sample size according to the level of inspection chosen and the size of the lot; Figure 1 provides the recommended sample size and acceptance number for the given AQL. For example, if the lot size is 100 and the normal level of inspection level II is chosen, the sample size code letter from Table 1 is F. Figure 1 recommends that a random sample of size 20 be taken. If the AQL is 4%, Figure 1 also shows that the lot should be accepted when there are 2 or fewer defective items, and rejected when there are more than 2 defective items in the sample.*

* The standard, in situations where a supplier submits a series of lots for inspection, also provides simple rules for a shift from normal (level II) to the tighter inspection level III whenever a large percentage of previously submitted lots were rejected, or to the reduced inspection level I whenever a large percentage of previously submitted lots were accepted. Rules for shifts in the opposite direction are also given. The standard offers the choice of three types of sampling plans: single, double, multiple. We describe here only single plans.

Sample size code letter	Sample size	Acceptable Quality Levels (normal inspection). 0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	↑	↑	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100 percent inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

Figure 1

Sample sizes and acceptance numbers

Negotiating a plan

Despite the widespread use of the standard, there is some ignorance about its properties. Often two private parties agree to abide by the recommended plan in the belief that the plan is “fair”, that is, treats buyer and seller equitably. That may not be so. One of the parties is often exposed to a considerably greater risk, as we shall demonstrate below.

In general, a sampling plan is completely specified by two numbers: the sample size, n, and the acceptance number, c. The plan calls for selecting a random sample of size n from the lot, for accepting the lot if the number of defectives in the sample, F, is less than or equal to c, and rejecting the lot if the number of defectives is greater than c.

One of two decisions, then, will be made: accept or reject the lot. Whether this decision is correct or incorrect depends on the unknown lot fraction defective, π. If π is less than or equal to the acceptable quality level, π_0, the lot would be considered acceptable (“Good”). If, on the other hand, π is greater than π_0, the lot would be unacceptable (“Bad”).

If the lot is good and it is accepted, or if it is bad and it is rejected, no error is made. If the lot is good but is rejected, an error is made; an error is also made if a bad lot is accepted. The situation is outlined in Table 2.

Table 2
States, decisions, and consequences

	Condition of lot	
Decision	Good ($\pi \leq \pi_0$)	Bad ($\pi > \pi_0$)
Accept ($F \leq c$)	No error	Error– –buyer concerned
Reject ($F > c$)	Error– –seller concerned	No error

These errors, however, do not have the same consequences. Rejection of a good lot is the sort of error that a seller is more anxious to avoid. Acceptance of a bad lot, on the other hand, is the sort of error that a buyer is anxious to avoid. The probability of rejecting a good lot under a specified sampling plan is called the “seller's risk”. Likewise, the probability of accepting a bad lot is called the “buyer's risk”.

Now, the distribution of the number of defective items (F) in a random sample of size n without replacement from a population of N items of which $100\pi\%$ are defective is hypergeometric with parameters N, n, and $k = N\pi$. (If sampling is with replacement, the probability distribution of F is binomial with parameters n and π.) The probability that the lot will be accepted is the probability that the number of defectives in the sample will be less than or equal to c. The probability that the lot will be rejected is the probability that the number of defectives in the sample will be greater than c. These probabilities depend on the unknown lot fraction defective, π. For example, if $\pi = 0.08$, $N = 100$, $n = 20$, and $c = 2$, the probability of accepting the lot is

$$Pr(F \leq 2|N = 100, n = 20, k = 8),$$

where, again, F is the number of defective items in the sample. This probability is equal to 0.804 (calculated by a computer program). The probability of rejecting the lot is equal to 1 minus the probability of acceptance, or 0.196.

Table 3
Probabilities of acceptance (A) and rejection (R)

Lot % defective	Lot quality*	$n = 20, c = O$		$n = 20, c = 1$		$n = 20, c = 2$		$n = 20, c = 3$	
		A	R	A	R	A	R	A	R
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1.0	G	0.7999	*0.2001*	1.0000	*0.0000*	1.0000	*0.0000*	1.0000	*0.0000*
2.0	G	0.6383	*0.3617*	0.9611	*0.0389*	1.0000	*0.0000*	1.0000	*0.0000*
3.0	G	0.5082	*0.4918*	0.8991	*0.1009*	0.9931	*0.0069*	1.0000	*0.0000*
4.0	G	0.4036	*0.5964*	0.8225	*0.1775*	0.9756	*0.0244*	0.9988	*0.0012*
5.0	B	**0.3191**	0.6809	**0.7396**	0.2604	**0.9470**	0.0530	**0.9949**	0.0051
6.0	B	**0.2518**	0.7482	**0.6546**	0.3454	**0.9069**	0.0931	**0.9854**	0.0146
7.0	B	**0.1986**	0.8014	**0.5737**	0.4263	**0.8588**	0.1412	**0.9715**	0.0285
8.0	B	**0.1558**	0.8442	**0.4975**	0.5025	**0.8041**	0.1959	**0.9512**	0.0488
9.0	B	**0.1218**	0.8782	**0.4266**	0.5734	**0.7442**	0.2558	**0.9241**	0.0759
10.0	B	**0.0952**	0.9048	**0.3629**	0.6371	**0.6811**	0.3189	**0.8905**	0.1095

*G = "Good" lot; B = "Bad" lot.
"Seller's risk" in *italics*, "buyer's risk" in **bold**.

Columns (7) and (8) of Table 3 show the probabilities of accepting and rejecting the lot for selected values of π when the sampling plan $(n = 20, c = 2)$ recommended by the standard is used. (Since the number of defectives in a lot can take only integer values $0, 1, 2, \ldots, N$, the possible values of π when $N = 100$ are $0, 0.01, 0.02, \ldots, 1.00$.)

The probability of rejecting a good lot (the seller's risk) is the probability of rejection when π is less than or equal to 0.04; these probabilities are shown in Table 3 in *italics*. The probability of accepting a bad lot (the buyer's risk) is the probability of acceptance when $\pi > 0.04$; these probabilities are shown in **bold** in Table 3. Table 3 reveals that, under the sampling plan recommended by the standard $(n = 20, c = 2)$, the maximum seller's risk is 2.4%. By contrast, the maximum buyer's risk is 94.7%. Obviously, this particular plan overwhelmingly favors the seller.

A buyer should not knowingly agree to such a plan, if he has some choice. Table 3 shows the risks associated with three other sampling plans: $(n = 20, c = 0)$, $(n = 20, c = 1)$, and $(n = 20, c = 3)$. Other plans can be examined in a similar fashion.

Of the four plans examined in Table 3, the buyer should prefer $(n = 20, c = 0)$. The seller should prefer $(n = 20, c = 3)$. Which plan is actually chosen depends entirely on the negotiating skill and knowledge of the two parties.

FOR ADDITIONAL INFORMATION

D. C. Montgomery, *Introduction to Statistical Quality Control,* 3rd ed., Wiley, 1995.

PROBLEMS

1: In the manner of Table 3:

(a) Determine the probabilities of acceptance and rejection for the sampling plans $(n = 20, c = 0)$, $(n = 20, c = 1)$, $(n = 20, c = 2)$, and $(n = 20, c = 3)$ when the AQL is 6%. Assume $N = 100$.

(b) Plot the buyer's and seller's risk as a function of π.

(c) Determine which of these sampling plans is such that the probability of accepting a bad lot does not exceed 50%. Which party does this plan favor?

2: A buyer and a seller agree to use the standard for testing the quality of a lot. The lot size is 300, the acceptable quality level is 1%, and the "normal" level (II) of inspection will be used.

(a) Determine the sampling plan to be used from Tables 1 and Figure 1.

(b) Calculate the maximum buyer's and seller's risks under this plan. Which party does this plan favor?

3: Consider the following simple sampling plan. Sample $100q\%$ of the items in a lot of N items for which the acceptable quality level is π_0. (The sample size is $n = Nq$.) Accept the lot if the fraction of defective items in the sample is less than or equal to π_0, and reject the lot if the fraction defective is greater than π_0. Analyze this sampling plan. Describe its properties and illustrate its features for selected values of N, q, and π_0.

4: A buyer and a seller agree to use the standard for testing the quality of a lot. The lot size is 10, the AQL is 25%, and General Inspection Level II will be used.

(a) Determine the sampling plan recommended by the standard.

(b) Calculate the maximum buyer's and seller's risk under this plan. Whom does this plan favour?

(c) Which of the two sampling plans: $(n = 3, c = 1)$ and $(n = 3, c = 2)$ is more equitable?

5: In preparation of negotiating a mutually acceptable sampling plan for testing the quality of a lot, a buyer and a seller used a computer program to calculate the probability of *rejecting* a lot for various sampling plans and parameter values.

The lot consists of 200 items. The acceptable quality level is 1%. The following table shows the probability of rejecting the lot:

$$Pr(F > c|N, n, k = N\pi),$$

where c is the acceptance number, $N = 200$ is the lot size, n the sample size, and π the proportion of defectives in the lot. Since sampling is without replacement, these probabilities are based on the hypergeometric distribution.

π	$n = 20$, $c = 0$	$n = 30$, $c = 0$	$n = 40$, $c = 0$	$n = 50$, $c = 0$	$n = 60$, $c = 0$
0.005 (.5%)	0.1004	0.1488	0.2002	0.2501	0.3001
0.010 (1%)	0.1909	0.2772	0.3618	0.4389	0.5113
0.015 (1.5%)	0.2722	0.3876	0.4907	0.5802	0.6596
0.020 (2%)	0.3460	0.4805	0.5935	0.6867	0.7628
0.025 (2.5%)	0.4131	0.5603	0.6764	0.7667	0.8355

(a) Determine the seller's and buyer's risk for each value of π and sampling plan. Determine the maximum seller's and buyer's risk for each sampling plan.

(b) Which of the sampling plans shown would the buyer prefer? The seller? Which sampling plan is the most equitable?