ARE STOCK PRICES PREDICTABLE?

by
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For some years now, the question of whether the history of a stock's price is relevant, useful or profitable in forecasting the future price of the stock has been a subject of controversy among academics and stock market professionals.

On the one hand, there are those who believe that stock prices tend to follow certain patterns. These patterns may be simple or complex, easy or difficult to identify, but are nonetheless predictable. Careful study of past prices, it is claimed, may reveal these patterns, which can then be used to forecast future prices, thereby providing profits for traders who buy or sell on the basis of the forecasts.[1]

On the other hand, there are some who argue that stock prices are no more predictable than the outcomes of a series of tosses of a coin, rolls of a die, or spins of a roulette wheel. In the stock market, proponents of this view say, the price of a stock is determined by its demand and supply. These are influenced by traders' expectations of the future earnings of the company. A change in the price of a stock will occur as a result of new information becoming available related to the future earnings of the company. Since this information is unlikely to have any connection to past prices, the study of the past should be of no value to the market analyst or investor—their efforts might more enjoyably be devoted to another pastime. This view has become known as the *random walk theory* of stock market prices.

To get a grasp of the issues, let us consider how an extreme—and rather outrageous—version of the random walk theory would operate.

Let us suppose that the closing price of a stock is in fact determined by someone with the help of a roulette wheel divided into three sections marked "−1," "0" , and "+1," as shown in Figure 1.

At the end of a business day, the wheel is spun and the section coming to rest against the pointer is noted. If it is the section labeled "0," this is interpreted to mean that the price did not change. If the section labeled "−1" rests against the pointer, the price change is \$−1, while the "+1" is interpreted as a price increase of \$1.

Because the section labeled "0" takes up one-half of the wheel's circumference, and the other two sections one-quarter each, a \$0 change should occur in 50% of the spins, a \$−1 change in 25%, and a \$+1 change in the remaining 25% of the spins. To illustrate, suppose that the wheel is spun 10 times, simulating 10 successive price

[1] The term *technical analysis* refers to this approach. Its followers tend to look at charts of past stock prices and trading volumes for clues concerning future prices. See, for example, Copsey (1999), Bauer and Dahlquist (1999), and Tadian (1996). An entertaining account of the controversy can be found in Malkiel (1985).

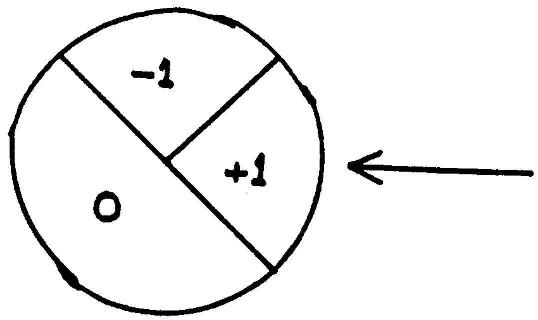

Figure 1
Partitioned roulette wheel

changes:

Day (t)	1	2	3	4	5	6	7	8	9	10
Price change (Y_t)	0	−1	0	1	1	0	0	0	−1	0

If the initial price of the stock was $10, the closing price of the stock at the end of each day would be:

Day (t) :	1	2	3	4	5	6	7	8	9	10
Closing price (X_t):	10	9	9	10	11	11	11	11	10	10

The hypothesis is, of course, preposterous, but the point is that *if* this were indeed the mechanism generating stock prices, a study of past price changes would be useless, since the outcome of any one roulette spin is unrelated to the outcome of any preceding or succeeding spin. The outcomes are *independent* of one another.

The random walk theory asserts that successive changes in the price of a stock behave *as if* they are generated by repeated spins of an appropriately designed roulette wheel, i.e., a wheel so partitioned as to reflect a realistic distribution of price changes.

Roberts (1959) carried out a simulation of weekly changes of a stock market index. Figure 2 shows 52 simulated index changes. These changes can be thought of as having been generated by a roulette wheel partitioned according to a certain distribution of changes of the index. Assuming that the initial level of the index was 450, the corresponding simulated index levels are shown in Figure 3.

It is interesting to note that Figure 3 *looks like* the chart of a stock market index. To an observer unaware of the manner in which it was constructed, it may even suggest a pattern and raise the hope of a profitable strategy. It may appear, for example, that positive changes tend to be followed by positive changes (weeks 8-30, 43-49), and that negative changes tend to be followed by negative changes (weeks 3-8, 30-43). If this were a stock rather than an index, a possible strategy

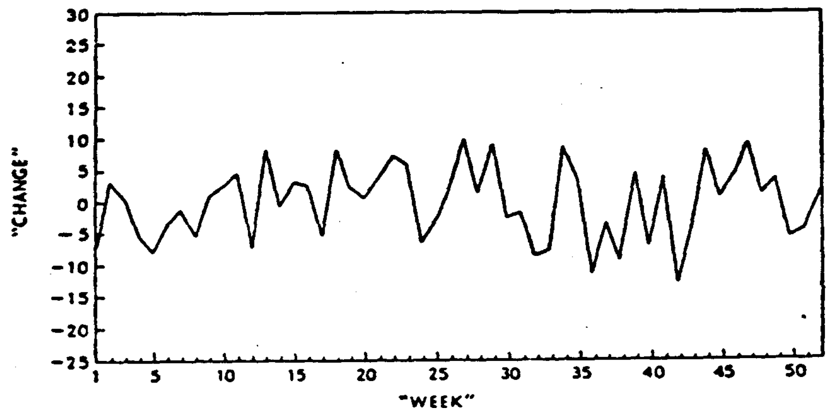

Figure 2
Simulated index changes for 52 weeks

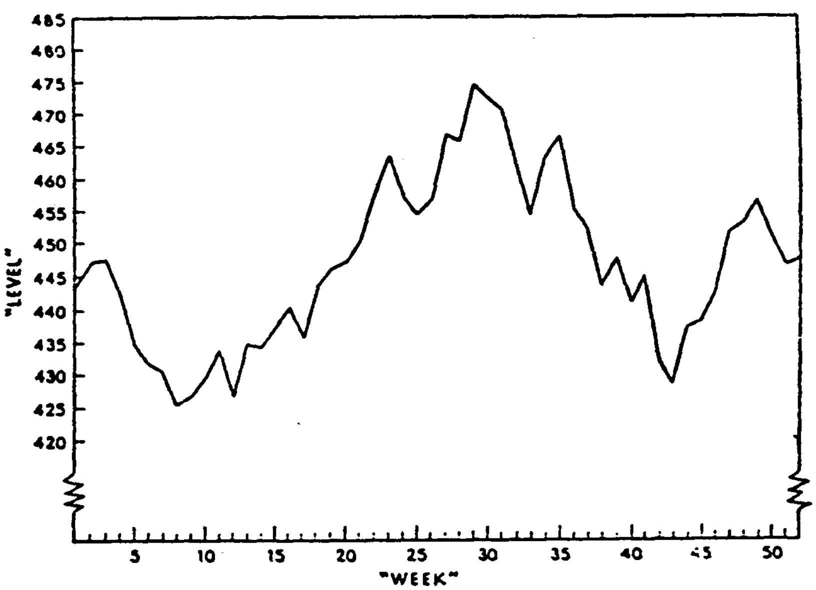

Figure 3
Simulated index levels for 52 weeks

might be to buy when the price just begins to rise and to sell when the price just begins to decline. Such a strategy may have worked for this particular series, but any resulting profit would have been accidental: in Roberts' simulation, in fact, a positive index change occurs 50% of the time and a negative one 50% of the time, regardless of whether the previous change was positive or negative.

Let us now consider *how to test* the random walk theory, that is, how to determine if changes in the price of a stock are independent of one another. Suppose that the *observed* closing price and price change of a certain stock on each of 10 consecutive trading days was as follows:

Day (t):	1	2	3	4	5	6	7	8	9	10
Closing price (X_t):	15	14	14	15	16	16	16	16	15	14
Price change, ($Y_t = X_t - X_{t-1}$):		-1	0	$+1$	$+1$	0	0	0	-1	-1

We may start with consecutive price changes and treat the eight pairs of changes: $(-1, 0)$, $(0, +1)$, ... as observations from a joint distribution, thereby forming the joint frequency distribution shown in Table 1.

Table 1
Joint frequency distribution of consecutive price changes

Today's change (Y_t)	Tomorrow's change (Y_{t+1}) -1	0	$+1$	Total
$+1$	0	1	1	2
0	1	2	1	4
-1	1	1	0	2
Total	2	4	2	8

For example: a price change of $+1$ was followed by a change of $+1$ once; a change of 0 was followed by change of 0 twice; and so on. From this joint frequency distribution, we construct the conditional distributions of tomorrow's change given today's change and the joint relative frequency distribution, as shown in Tables 2 and 3.

Now, if tomorrow's price change was independent of today's change, the distributions of tomorrow's change given that today's change is $+1$ (row 1 of Table 2) or 0 (row 2), or -1 (row 3) should be identical. Equivalently, if today's change and tomorrow's change were independent of one another, the joint relative frequencies of Table 3 should be equal to the product (shown in parentheses) of the marginal relative frequencies.

In this artificial example, the strict definition of independence is not satisfied. The number of observations is, of course, far too small to support any reliable conclusions. However, even if a reasonably large number of observations were available,

Table 2
Conditional distributions
of tomorrow's price change

Today's change (Y_t)	Tomorrow's change (Y_{t+1}) -1	0	$+1$	Total
$+1$	0	1/2	1/2	1
0	1/4	1/2	1/4	1
-1	1/2	1/2	0	1

Table 3
Joint relative frequency distribution of today's and
tomorrow's price change

Today's change (Y_t)	Tomorrow's change (Y_{t+1}) -1	0	$+1$	Total
$+1$	0 (1/16)	1/8 (1/8)	1/8 (1/16)	1/4
0	1/8 (1/8)	2/8 (2/8)	1/8 (1/8)	1/2
-1	1/8 (1/16)	1/8 (1/8)	0 (1/16)	1/4
Total	1/4	1/2	1/4	1

Note: Numbers in parentheses are the products of the marginal relative frequencies.

we would not expect the strict definition of statistical independence to be satisfied exactly. For practical purposes, we can treat two variables as independent if the definition of statistical independence is *approximately* satisfied.

Of course, in addition to (or instead of) a relationship between consecutive price changes, there may be a *lagged* relationship between price changes—tomorrow's change may be related to yesterday's change, or to the change two days ago, etc. To illustrate, let us consider the relationship between price changes lagged two days. Using the same series of price changes,

Day (t):	1	2	3	4	5	6	7	8	9	10
Price change (Y_{t-1}):		-1	0	$+1$	$+1$	0	0	0	-1	0

and pairs of changes lagged two days: $(-1,+1)$, $(0,+1)$, $(+1,0)$, …, we get the joint frequency distribution shown in Table 4. We may now proceed exactly as in the previous case to examine if the two variables are independent of one another.

The same approach may be used to examine the relationship between price changes lagged three days, four days, and so on.

Table 4
Joint frequency distribution of price changes lagged two days

Today's change (Y_t)	Tomorrow's change (Y_{t+1}) −1	0	+1	Total
−1	0	0	1	1
0	2	1	1	4
+1	0	2	0	2
Total	2	3	2	7

This procedure for determining whether independence holds is not too cumbersome when only one time series is examined for one particular form of dependence. As the number of time series, numerical values, and lags examined becomes larger, the need for a summary measure becomes stronger. The correlation coefficient (r), it will be recalled, is a summary measure of the extent of a linear relationship between two variables. If the variables are independent, r equals 0; however, r equals 0 also in some cases where the variables are related but in a non-linear fashion. Thus, in using the correlation coefficient as a measure of dependence there is a risk of reaching the wrong conclusion, but the convenience of having a summary measure for a large number of joint distributions outweighs by far the slight risk involved.

In one of the earliest and comprehensive studies of stock market prices, Fama (1965) analyzed the behavior of daily price changes for each of the thirty stocks of the Dow-Jones Industrial Average. The time periods varied from stock to stock. There were, in all, thirty time series, each with about 1,200 to 1,700 observations. For each stock, Fama calculated ten correlation coefficients, summarizing the relationship between daily price changes lagged 1, 2, ..., 9, and 10 days; these are shown in Figure 4.

All the correlation coefficients shown in Figure 4 are quite small in absolute value, indicating that little, if any, relationship exists between consecutive or lagged daily changes, or between consecutive changes across intervals of more than one day. Correlation coefficients as close to 0 as these appear to support the hypothesis that stock price changes are independent of one another.[2]

Numerous subsequent studies [see, for example, the bibliography in Elton and

[2] Actually, Fama's price change is not the arithmetic difference between daily prices, but the difference in the natural logarithms of these prices. If X_t denotes the price on day t, the arithmetic difference is $X_t - X_{t-1}$, while the logarithmic difference is $\log X_t - \log X_{t-1} = \log(X_t/X_{t-1})$. The main reason for using changes in the logarithm of prices, rather than ordinary price changes, is that the variability of ordinary price changes tends to depend on the price level of the stock while that of logarithmic changes does not. Although it appears rather awkward, the change

Stock	Lag									
	1	2	3	4	5	6	7	8	9	10
Allied Chemical....	.017	−.042	.007	−.001	.027	.004	−.017	−.026	−.017	−.007
Alcoa............	.118	.038	−.014	.022	−.022	.009	.017	.007	−.001	−.033
American Can.....	−.087	−.024	.034	−.065	−.017	−.006	.015	.025	−.047	−.040
A.T.&T..........	−.039	−.097	.000	.025	.005	−.005	.002	.027	−.014	.007
American Tobacco..	.111	−.109	−.060	−.065	.007	−.010	.011	.046	.039	.041
Anaconda.........	.067	−.061	−.047	−.002	.000	−.038	.009	.016	−.014	−.056
Bethlehem Steel...	.013	−.065	.009	.021	−.053	−.098	−.010	.004	−.002	−.021
Chrysler..........	.012	−.066	−.016	−.007	−.015	.009	.037	.056	−.044	.021
Du Pont..........	.013	−.033	.060	.027	−.002	−.047	.020	.011	−.034	.001
Eastman Kodak....	.025	.014	−.031	.005	−.022	.012	.007	.006	.003	.002
General Electric....	.011	−.038	−.021	.031	−.001	.000	−.008	.014	−.002	.010
General Foods.....	.061	−.005	.045	.002	−.015	−.052	−.006	−.014	−.024	−.017
General Motors....	−.004	−.056	−.037	−.008	−.038	−.006	.019	.006	−.016	.009
Goodyear.........	−.123	.017	−.044	.043	−.002	−.003	.035	.014	−.015	.007
International Harvester..........	−.017	−.029	−.031	.037	−.052	−.021	−.001	.003	−.046	−.016
International Nickel	.096	−.033	−.019	.020	.027	.059	−.038	−.008	−.016	.034
International Paper.	.046	−.011	−.058	.053	.049	−.003	−.025	−.019	−.003	−.021
Johns Manville....	.006	−.038	−.027	−.023	−.029	−.080	.040	.018	−.037	.029
Owens Illinois......	−.021	−.084	−.047	.068	.086	−.040	.011	−.040	.067	−.043
Procter & Gamble..	.099	−.009	−.008	.009	−.015	.022	.012	−.012	−.022	−.021
Sears.............	.097	.026	.028	.025	.005	−.054	−.006	−.010	−.008	−.009
Standard Oil (Calif.)	.025	−.030	−.051	−.025	−.047	−.034	−.010	.072	−.049	−.035
Standard Oil (N.J.).	.008	−.116	.016	.014	−.047	−.018	−.022	−.026	−.073	.081
Swift & Co........	−.004	−.015	−.010	.012	.057	.012	−.043	.014	.012	.001
Texaco...........	.094	−.049	−.024	−.018	−.017	−.009	.031	.032	−.013	.008
Union Carbide.....	.107	−.012	.040	.046	−.036	−.034	.003	−.008	−.054	−.037
United Aircraft....	.014	−.033	−.022	−.047	−.067	−.053	.046	.037	.015	−.019
U.S. Steel.........	.040	−.074	.014	.011	−.012	−.021	.041	.037	−.021	−.044
Westinghouse......	−.027	−.022	−.036	−.003	.000	−.054	−.020	.013	−.014	.008
Woolworth........	.023	−.016	.015	.014	.007	−.039	−.013	.003	−.088	−.008

Figure 4
Correlation coefficients, Fama study

in log price can be used very much like the ordinary price change. Given an initial price, X_t , the price on day $t+2$, say, can be reproduced either by means of ordinary price changes

$$X_{t+2} = X_t + (X_{t+1} - X_t) + (X_{t+2} - X_{t+1}),$$

or by means of log price changes

$$\log X_{t+2} = \log X_t + (\log X_{t+1} - \log X_t) + (\log X_{t+2} - \log X_{t+1}),$$

from which X_{t+2} can be obtained.

Gruber (1995, pp. 440-8)] arrived at similar conclusions: the correlation of consecutive and lagged price changes tendes to be very low. It is conceivable, of course, that a trading policy could be devised that would take advantage of even such low correlation.[3] To this date, however, it has yet to be demonstrated that a trading policy exists yielding consistently better profits after commissions and other expenses than a simple "buy and hold" policy.

Proponents of the random walk theory conclude that knowledge of the history of the price of a stock is of no practical value in forecasting the future price of the stock.

This, it should be emphasized, does not mean that an accurate stock price forecast (hence also profit) cannot be made. The price of a stock changes continuously as new information affecting the future profits of the firm becomes available. Traders who have or anticipate this new information, and evaluate correctly its effects upon the future profits of the firm are likely to make greater profits than traders without this information. Their advantage, however, lies in the new information they possess, not in their study of past prices.

PROBLEMS

1: Niederhoffer and Osborne (1966) examined the distribution of changes in the price of a number of stocks at *consecutive transactions* (not at the daily close, as in the text of this reading). Their findings may be summarized approximately as in Table 5.

Table 5
Relative frequency distribution of
consecutive pairs of price changes

	"Next" change			
"This" change	Negative	Zero	Positive	Total
Negative	0.03	0.11	0.09	0.23
Zero	0.12	0.30	0.11	0.53
Positive	0.09	0.12	0.03	0.24
Total	0.24	0.53	0.23	1.00

For example, in 12% of the pairs of consecutive price changes examined, a zero

[3] See Problem 2 for two simple trading policies. In the literature of finance, there is a large number of studies that examine the performance of a variety of trading policies against the prices that actually occurred. Three of the early studies, for example, are S. S. Alexander, "Price movements in speculative markets; trends and random walks, no. 2" in Cootner (1964); E. F. Fama, "Efficient capital markets: a review of theory and empirical work," and M. C. Jensen and G. A. Bennington, "Random walks and technical theories," both in Lorie and Brealey (1972).

price change was followed by a negative price change; in 30% of the pairs, the price changes were both equal to zero; and so on.

(a) Determine the conditional distributions of the "next" change given that "this" change is negative, zero, or positive. Interpret these distributions.
(b) Are consecutive transaction price changes independent? Why?
(c) Discuss the implications of your answers to (a) and (b).

2: It is conceivable that a trading policy could be devised that would take advantage of even the low degree of dependence observed in empirical studies. Consider one simple trading policy:

> Set aside a certain amount of money. Buy as many shares as you can afford when the price rises. Hold the shares as long as the price continues to rise, and sell all shares when the price falls. Continue buying and selling in this manner until the cumulative gain or loss exceeds x% of the starting capital, at which time liquidate any shares held and stop trading.

(a) Find out how well this trading policy would have performed had it been implemented in the past, given the following observed sequence of the price a certain stock:

Day	Price ($)	Day	Price ($)	Day	Price ($)	Day	Price
1	22	6	25	11	23	16	20
2	27	7	24	12	28	17	28
3	28	8	23	13	28	18	25
4	24	9	26	14	29	19	25
5	25	10	28	15	26	20	24

Assume the starting price is $27, the initial capital is $1,000, that a commission of 1% must be paid on the value of any transaction, and that $x = 40\%$.

(b) Same as (a) except the sequence of prices is as follows:

Day	Price ($)	Day	Price ($)	Day	Price ($)	Day	Price
1	21	6	26	11	27	16	26
2	22	7	27	12	26	17	29
3	23	8	28	13	25	18	28
4	24	9	29	14	26	19	27
5	25	10	28	15	27	20	26

(c) Comment on your findings in (a) and (b).

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