There is a strong analogy:

$$\text{function} \longleftrightarrow \text{vector}$$
$$\text{operator} \longleftrightarrow \text{matrix}$$

We know $f(x)$ if we know its value for every $x$. In practice, one can say we *know* a function $f(x)$ if we know its value for *many* points $x$ that span the range we're interested in.

Suppose we evaluate $f(x)$ at many points $x$, $x = \ldots, 0.00$, 0.01, 0.02, 0.03, ...and we arrange the values $\ldots f(0.00)$, $f(0.01)$, $f(0.02)$, $f(0.03)$, ...in a vector. Let's call that vector $\vec{f}$ and represent it as:

$$\begin{pmatrix} \ldots \\ \mathrm{f}(0.00) \\ \mathrm{f}(0.01) \\ \mathrm{f}(0.02) \\ \mathrm{f}(0.03) \\ \ldots \end{pmatrix}$$

We can calculate the integral on the left-hand side below (the so-called *overlap* integral of $f(x)$ and $g(x)$) to a good approximation using vectors:

$$
\begin{aligned}
\int f(x) \;\; g(x)\, dx = 0.01 \times ( \ldots \\
+ \quad & f(0.00) \cdot g(0.00) + f(0.01) \cdot g(0.01) \\
+ \quad & f(0.02) \cdot g(0.02) + f(0.03) \cdot g(0.03) \\
+ \quad & \ldots ) \\
= \quad & 0.01 \times \vec{f} \cdot \vec{g}
\end{aligned}
$$

Integrals of products of functions are just like *scalar products of vectors* !

*Functions are like vectors with an infinite number of components.*

quantum mechanics / *matrix mechanics* / *wave mechanics*

The analogy goes much further. In matrix-vector algebra, one often uses a set of *unit length orthogonal vectors* as a basis for representing other vectors (see next page). Likewise, one can use a set of *orthonormal functions* as a basis for representing other functions.

Complete orthonormal set of vectors in 3D space:

$$\vec{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} , \; \vec{u}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} , \; \vec{u}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

For any 3D vector we can write

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = c_1\vec{u}_1 + c_2\vec{u}_2 + c_3\vec{u}_3 = \sum_{n=1}^{3} c_n\vec{u}_n$$

Many choices of orthonormal basis set of vectors are possible, for ex.:

$$\vec{u}'_1 = \begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{pmatrix} , \; \vec{u}'_2 = \begin{pmatrix} \sqrt{2}/2 \\ -\sqrt{2}/2 \\ 0 \end{pmatrix} , \; \vec{u}'_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

for which we can write

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = c'_1\vec{u}'_1 + c'_2\vec{u}'_2 + c'_3\vec{u}'_3 = \sum_{n=1}^{3} c'_n\vec{u}'_n$$