

Functions, Operators  $\hat{O}$ , and Functionals

A function  $f(x)$  operates on a number  $x$  to produce another number  $y = f(x)$ :

$$x \xrightarrow{f} y$$

An operator  $\hat{O}$  operates on a function  $f(x)$  to produce another function  $g(x) = \hat{O}f(x)$ :

$$f(x) \xrightarrow{\hat{O}} g(x)$$

A functional  $\mathcal{F}$  operates on a function  $f(x)$  to produce a number  $y$

$$f(x) \xrightarrow{\mathcal{F}} y$$

Operators:  $\hat{O}\psi(x)$

$$\frac{d^2}{dx^2}(\quad) \quad \frac{d^2}{dx^2}3x^2 = 6x$$

$$(\quad)^2 \quad (3x^2)^2 = 9x^4$$

$$2x \cdot (\quad) \quad 2x \cdot (3x^2) = 6x^3$$

Functionals:  $\mathcal{F}[\psi(x)]$

$$\int_{-1}^{+1} (\quad) dx \quad \int_{-1}^{+1} (x^2) dx = 2/3$$

$$\text{Min}(\quad) \quad \text{Min}(x^2) = 0$$

$$\left. \frac{d(\quad)}{dx} \right|_{x=1} \quad \left. \frac{d(x^2)}{dx} \right|_{x=1} = 2$$