(4) Know the <u>type</u> and <u>range</u> of the variables and functions in the equation

$$x^2 + x + 1 = 0 \qquad (1)$$

$$x\,(x+1) + 1 = 0 \qquad (2)$$

$$(x+1) = -1/x \qquad (3)$$

$$(3) \text{ in } (1): \quad x^2 + (-1/x) = 0 \qquad (4)$$

$$x^2 = 1/x \qquad (5)$$

$$x^3 = 1 \quad \Rightarrow \quad x = 1 \qquad (6)$$

Substitute in (1):

$$1^2 + 1 + 1 = 0$$

In solving for x, we forgot that:

(1) In the original equation, x stands for one of **two** possible unknown numbers;
but in Eqn. (6), x stands for one of **three** possible unknown numbers, it's not the same thing anymore!

(2) x is a complex number, it may or may not be real.

(5) Dimensional analysis helps

Kinetic energy operator:

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} \;:\; (J\cdot s)^2\, kg^{-1}\, m^{-2}$$

$$:\; (kg\, m^2\, s^{-2}\, s)^2\, kg^{-1}\, m^{-2}$$

$$:\; kg^2\, m^4\, s^{-2}\, kg^{-1}\, m^{-2}$$

$$:\; kg\, m^2\, s^{-2} \;:\; J$$

$$K = \int \psi(x)^* \left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\right) \psi(x)\, dx$$

...

...

Other example, kinetic energy operator for an electron, in atomic units:

$$\hat{K} = \frac{-1}{2}\frac{d^2}{dx^2}$$

Looks like m^{-2}, but it *really* is J because

$\hbar = 1$ *a.u. of action* and $m_e = 1$ *a.u. of mass*, so

$\hbar^2/m_e$ which is omitted is "1 a.u. of action^2/mass", not "1".

$$0 = -\left(\frac{\hbar^2}{2\mu}\right)\frac{d^2}{dr^2}S_{n,\ell} + \left[\frac{\hbar^2\ell(\ell+1)}{2\mu r^2} - \frac{Ze^2}{4\pi\epsilon_0 r} - E\right]S_{n,\ell}$$

$\hbar^2, e^2, 4\pi\epsilon_0$: fundamental constants

μ, Z: known constants for a given system

r: variable, goes from 0 to infinity

n, ℓ: quantum numbers, integers; different QN for different solutions to the Schrödinger equation

E: unknown constant, energy for a state with given n, ℓ.

$S_{n,\ell}$: a function of r. It is real; can be positive or negative.

Good habits:

- Make sense of equations, with visual things if possible (plots, sketches, analogies)
- Do not give too much importance to symbols
- Classify the things in each equation as: variables, constants, functions, operators, ...
- Do dimensional analysis
- Think about the context for each equation
- Think about the range of variables
- Think about limiting values
- Think about proportionality relations