Problem 1. (4 pts)

a) Convert $101101_2$ from binary to decimal.

1
0 $2 \times 1 + 0 = 2$
1 $2 \times 2 + 1 = 5$
1 $2 \times 5 + 1 = 11$
0 $2 \times 11 + 0 = 22$
1 $2 \times 22 + 1 = \boxed{45_{10}}$

b) Convert $421_{10}$ from decimal to octal (base 8).

$\times \frac{1}{8}$

$421 = 8 \times 52 + 5$
$52 = 8 \cdot 6 + 4$
$6 = 8 \cdot 0 + 6$

$\boxed{645_8}$

c) Convert $101101110101_2$ from binary to hexadecimal.

$B\ 7\ 5_{16}$

d) Convert $3721_8$ from octal to binary.

$\boxed{011\ 111\ 010\ 001_2}$

Problem 2. (4 pts)

Use Mathematical Induction to prove that

$$1 \cdot 1! + 2 \cdot 2! + \cdots n \cdot n! = (n+1)! - 1$$

1pt

Base Step $n=1$ $\quad 1\cdot 1! = 2! - 1 = 1$ ✓

Induction Step

Assume

$$1\cdot 1! + 2\cdot 2! + \cdots + n\cdot n! = (n+1)! - 1$$

Add $(n+1)(n+1)!$ $\Rightarrow$ $1\cdot ! + \cdots + n\cdot n! + (n+1)(n+1)! = (n+1)! - 1 + (n+1)(n+1)!$

$$= (n+1)!\,(1+n+1) - 1$$

$$= (n+1)!\,(n+2) - 1$$

$$= (n+2)! - 1$$

Problem 3. (3 pts)

Construct an argument using the rules of inference to show that the hypotheses "There is someone in this class who has been to France" and "Everyone who has been to France visits the Lourve" imply the conclusion "Therefore, someone in this class has visited the Lourve". Use the predicates F(x) for "x has been to France" C(X) for " x is in this class" and L(x) for "x has visited the Lourve".

1) $\exists x\,(C(x) \wedge F(x))$ Hyp

2) $\forall x\,(F(x) \to L(x))$ Hyp

3) $C(s) \wedge F(s)$ Existential Instantiation from 1)

4) $F(s) \to L(s)$ Universal Instantiation from 2

5) $F(s)$ Simplification from 3)

6) $L(s)$ Modus Ponen

7) $C(s)$ Simplification from 3

8) $C(s) \wedge L(s)$ Conjunction from 6, 7

9) $\exists x\,(C(x) \wedge L(x))$ Existential Generalization from 8

Note it isn't necessary to name all these steps

Problem 4. (3 pts) (Leave your answers to the problems below in combinatorial form; do not work out factorials.)

a) How many bit strings of length twelve either begin with two 0's or end with three 1's?

Begin with 00 $\quad 2^{10}$

End with 111 $\quad 2^{9}$

Both 00—111 $\quad 2^{7}$

$\therefore$ total $\boxed{2^{10} + 2^{9} - 2^{7}}$

b) Suppose a department contains 10 men and 15 women. How many ways are there to form a committee with four members if it must have more women than men?

3 women, 1 man $\quad C(15,3) \cdot C(10,1)$

4 women $\quad C(15,4)$

Total $\quad C(15,3) \cdot C(10,1) + C(15,4)$

c) How many functions are there from the set $\{1, \cdots n\}$ to the set $\{1, \cdots m\}$ if both 1 and n must be assigned the same value? Assume $n \geq 2$

Choose value for 1 and $n$ in $m$ ways

Choose values for other $n-2$ in $m^{n-2}$ ways

total

$$m \cdot m^{n-2} = \boxed{m^{n-1}}$$

d) How many ways are there to choose a fifteen cookies from the 10 different types at a cookie store if at least four chocolate chip cookies must be choosen?

$n$ = number of types = 10

$r$ = number of choices = $15-4$ $= 11$ since 4 must be choc chip

$$C(10+11-1, 11) = C(20, 11)$$