Problem 1. (4 pts)

a) Convert 100111_2 from binary to decimal.

1

0 $2 \times 1 + 0 = 2$

0 $2 \times 2 + 0 = 4$

1 $2 \times 4 + 1 = 9$

1 $2 \times 9 + 1 = 19$

1 $2 \times 19 + 1 = \boxed{39_{10}}$

b) Convert 391_{10} from decimal to octal (base 8).

$\times \frac{1}{8}$

$391 = 8 \times 48 + 7$

$48 = 8 \times 6 + 0$

$6 = 8 \cdot 0 + 6$

$\boxed{607_8}$

c) Convert 110011010101_2 from binary to hexadecimal.

$\boxed{C\ D\ 5_{16}}$

d) Convert 4612_8 from octal to binary.

$\boxed{100\ 110\ 001\ 010_2}$

Problem 2. (4 pts)

Use Mathematical Induction to prove that

$$1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 \cdots n \cdot 2^{n-1} = (n-1)2^n + 1$$

1pt Base Step $n=1$ $\quad 1 \cdot 2^0 = 0 \cdot 2^0 + 1 \Rightarrow 1 = 1$ ✓

3pts Induction Step

Assume $1 \cdot 2^0 + \cdots + n \cdot 2^{n-1} = (n-1)2^n + 1$

Add $(n+1)2^n$ to both sides $\Rightarrow$

$$1 \cdot 2^0 + \cdots + n \cdot 2^{n-1} + (n+1)2^n = (n-1)2^n + 1 + (n+1)2^n$$
$$= 2^n((n-1) + (n+1)) + 1$$
$$= 2^n \cdot 2 \cdot n + 1$$
$$= n \cdot 2^{n+1} + 1$$

✓

Problem 3. (3 pts)

Construct an argument using the rules of inference to show that the hypotheses "There is someone at York who has been to Banff" and "Everyone who has been to Banff visits Lake Louise" imply the conclusion "Therefore, someone at York has visited Lake Louise". Use the predicates B(x) for "x has been to Banff" Y(x1) for " x is at York" and LL(x) for "x has visited the Lake Louise".

1) $\exists x \, (Y(x) \wedge B(x))$ Hypothesis

2) $\forall x \, (B(x) \rightarrow LL(x))$ Hypothesis

3) $Y(c) \wedge B(c)$ Existential Instantiation from 1

4) $B(c) \rightarrow LL(c)$ Universal Instantiation from 2

5) $B(c)$ Simplification from 3

6) $LL(c)$ Modus Ponen from 4, 5

7) $Y(c)$ Simplification from 3)

8) $Y(c) \wedge LL(c)$ Conjunction from 5) 7)

9) $\exists x \, (Y(x) \wedge LL(x))$ Existential generalization from 8

It is not necessary to name these steps

Problem 4. (3 pts) (Leave your answers to the problems below in combinatorial form; do not work out factorials.)

a) How many bit strings of length 16 either begin with one 1 or end with three 0's?

Begin with 1 — 2^{15}

End with 000 2^{13}

Both 2^{12}

total $2^{15} + 2^{13} - 2^{12}$

b) Suppose a department contains 15 men and 12 women. How many ways are there to form a committee with four members if it must have more women than men?

3 women, 1 man $C(12,3)\,C(15,1)$

All women $C(12,4)$

total $C(12,3) \cdot C(15,1) + C(12,4)$

c) How many functions are there from the set $\{1, \cdots n\}$ to the set $\{1, \cdots m\}$ if both 1 and n must be assigned different values? Assume $n \geq 2$

Choose values for 1, n in $m \times (m-1)$ ways

Choose values for n-2 others in m^{n-2} ways

total $m(m-1)\,m^{n-2} = m^{n-1}(m-1)$

d) How many ways are there to choose a fourteen cookies from the 17 varieties at a cookie store if at least six chocolate chip cookies must be choosen?

n = number of types = 17

r = number of Choices = $14 - 6 = 8$ Since 6 must be choc chip

$C(17+8-1, 8) = C(24, 8)$