Problem 1. (4 pts)

Show that $(\neg(p \to q)) \to p$ is a tautology :

2 pts

a) using a truth table

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
F	F	T	F	T
F	T	T	F	T
T	F	F	T	T
T	T	T	F	T

b) using logical equivalence.

2 pts

$(\neg(p \to q)) \to p$

$\Leftrightarrow (\neg(\neg p \vee q)) \to p$

$\Leftrightarrow (p \wedge \neg q) \to p$

$\Leftrightarrow \neg(p \wedge \neg q) \vee p$

$\Leftrightarrow (\neg p \vee q) \vee p$

$\Leftrightarrow p \vee (\neg p \vee q)$

$\Leftrightarrow (p \vee \neg p) \vee q$

$\Leftrightarrow T \vee q$

$\Leftrightarrow T$

Problem 2. (4 pts) State the converse and the contrapositive of the following statements:

a) "I will go swimming if it is sunny."

converse:

If I go swimming then it is sunny

1pt each

contrapositive:

If I don't go swimming then it is not sunny

b) " I stay indoors only if it rains"

converse:

If it rains then I stay indoors

contrapositive

If it doesn't rain then I don't stay indoors

Problem 3. (4 pts) Let $P(m,n)$ be $m \leq n$ where m and n are integers. What are the truth values of

a) $\forall n \exists m P(m,n)$

T

b) $\exists n \forall m P(m,n)$

F

c) Write the negation of b) without using the negation symbol or the word "not"

$\neg(\exists n \forall m\ P(m,n))$

$\Leftrightarrow \forall n \neg \forall m\ P(m,n)$

$\Leftrightarrow \forall n \exists m \neg P(m,n)$

$\Leftrightarrow \forall n \exists m \neg(m \leq n)$

$\Leftrightarrow \forall n \exists m\ (m > n)$

Problem 4. (4 pts)

Mark either true or false:

a) $\{a, b\} \subseteq \{\{b\}, \{a\}, \{a, b\}\}$

F

b) $\phi \in \{\{\phi\}\}$

F

c) $a \in \mathcal{P}(\{a, b\})$

F

d) $\phi \subseteq \{a, b, c\}$

T