

Problem 1. (4 pts)

Show that $(\neg(p \rightarrow q)) \rightarrow \neg q$ is a tautology by:

a) using a truth table

2 pts

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg q$	$(\neg(p \rightarrow q)) \rightarrow \neg q$
F	F	T	F	T	
F	T	T	F	F	
T	F	F	T	T	
T	T	T	F	F	T

b) using logical equivalence.

2 pts

$$\begin{aligned} & (\neg(p \rightarrow q)) \rightarrow \neg q \\ \Leftrightarrow & \neg(\neg p \vee q) \rightarrow \neg q \\ \Leftrightarrow & p \wedge \neg q \rightarrow \neg q \\ \Leftrightarrow & \neg(p \wedge \neg q) \vee \neg q \\ \Leftrightarrow & (\neg p \vee q) \vee \neg q \\ \Leftrightarrow & \neg p \vee (q \vee \neg q) \\ \Leftrightarrow & \neg p \vee T \\ \Leftrightarrow & T \end{aligned}$$

Problem 2. (4 pts) State the converse and the contrapositive of the following statements:

a) "I swim only if it is warm"

converse:

If it is warm then I swim

1 pt
each

contrapositive:

If it is not warm, then I don't swim

b) "Whenever it snows I go skiing"

converse:

~~If I don't go skiing then it isn't snowing~~

If I go skiing then it is snowing

contrapositive

If I don't go skiing, then it isn't snowing

Problem 3. (4 pts) Let $P(m, n)$ be $m \leq n$ where m and n are integers.
What are the truth values of

a) $\exists m \forall n P(m, n)$

1 pt

F

b) $\forall m \exists n P(m, n)$

1 pt

T

c) Write the negation of b) without using the negation symbol or the word "not"

2 pts

$$\neg (\forall m \exists n P(m, n))$$

$$\Leftrightarrow \exists m \neg \exists n P(m, n)$$

$$\Leftrightarrow \exists m \forall n \neg P(m, n)$$

$$\Leftrightarrow \exists m \forall n \neg (m \leq n)$$

$$\Leftrightarrow \exists m \forall n (m > n)$$

Problem 4. (4 pts)

Mark either true or false:

a) $\{a\} \in \{\emptyset, \{a\}\}$

T

b) $\emptyset \subseteq \{a, \{\emptyset\}\}$

T

c) $\emptyset \in \{a\}$

F

d) $\{a\} \in \mathcal{P}(\{a, b\})$

T