Prob Set - March 18

12.1 48) $a_1 = 11$, $11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1 \cdots$

The Collatz conjecture is that this sequence always reaches 1, regardless of the starting point

12.2 18) $\sum \frac{1}{e^{2n}} = \sum \left(\frac{1}{e^2}\right)^n$ cgt geometric series, with $a = \frac{1}{e^2} < 1$, sum $\frac{1}{e^2 - 1}$

22) $\sum_{n=1}^{\infty} \frac{3}{n} = 3 \sum \frac{1}{n}$ divergent (harmonic) series

24) $\sum \frac{(n+1)^2}{n(n+2)}$ Diverges, since $a_n = \frac{(n+1)^2}{n(n+2)} \to 1 \neq 0$

42) $\sum (x-4)^n$ geometric series, with $r = x-4$, converges if $|x-4| < 1$ $\iff 3 < x < 5$ with sum $\frac{x-4}{5-x}$

Prob Set - March 25

12.3 22) $f = \frac{\ln x}{x^2}$ continuous, positive, and $f' = \frac{1 - 2\ln x}{x^3} < 0$ for $x \geq 2$ so f decreasing

$\int_2^{\infty} \frac{\ln x}{x^2} dx = \lim_{t \to \infty} \left[-\frac{\ln x}{x} - \frac{1}{x} \right]_1^t = 1$ (by l'Hospital's rule) so $\sum \frac{\ln n}{n^2}$ converges

24) $\frac{1}{x \ln x \ln \ln x}$ is continuous, positive decreasing, $\int_3^{\infty} \frac{dx}{x \ln x \ln \ln x} = \lim_{t \to \infty} \ln(\ln(\ln x))\Big|_3^t = \infty$

So $\sum \frac{1}{n \ln n \ln \ln n}$ diverges

12.4 4) $\frac{2}{n^3+4} < \frac{2}{n^3}$ so $\sum \frac{2}{n^3+4}$ converges by comparison with $\sum \frac{2}{n^3} = 2 \sum \frac{1}{n^3}$, a convergent p series

8) $\sum \frac{4+3^n}{2^n}$ $\frac{4+3^n}{2^n} > \frac{3^n}{2^n} = \left(\frac{3}{2}\right)^n$ so $\sum \frac{4+3^n}{2^n}$ diverges by comparison with divergent geometric series $\sum \left(\frac{3}{2}\right)^n$

12.6 6) By ratio test

$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(-3)^{n+1}/(n+1)!}{(-3)^n / n!} \right| = 3 \lim_{n \to \infty} \frac{1}{n+1} = 0 < 1$ So series is absolutely convergent