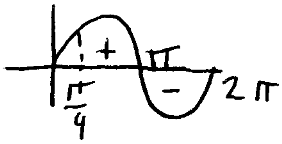
1310 - Term Test 1 Solutions

1 a) 

$$\int_{\pi/4}^{2\pi} |\sin x|\,dx = \int_{\pi/4}^{\pi} \sin x\,dx - \int_{\pi}^{2\pi} \sin x\,dx = -\cos x\Big|_{\pi/4}^{\pi} + \cos x\Big|_{\pi}^{2\pi}$$

$$= \left(1+\frac{\sqrt{2}}{2}\right) + (1+1) = \boxed{3+\frac{\sqrt{2}}{2}}$$

b) $\int_{-1}^{1} \frac{\tan x}{x^4+1}\,dx = \boxed{0}$ Since integrand is odd!

2) a) $\int \sec^2 x\sqrt{1+\tan x}\,dx$ Let $u=\tan x$, $du = \sec^2 x\,dx$

$$= \int \sqrt{1+u}\,du = \frac{2}{3}(1+u)^{3/2}+C = \boxed{\frac{2}{3}(1+\tan x)^{3/2}+C}$$

b) $\int x^3\sqrt{x^2-1}\,dx$ Let $u = x^2-1$ $du = 2x\,dx$

$x^2 = u+1$ $\frac{du}{2} = x\,dx$

$$= \int x^2\sqrt{x^2-1}\,x\,dx$$

$$= \int (u+1)\sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int\left(u^{3/2}+u^{1/2}\right)du = \frac{1}{2}\left(\frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}\right)+C$$

$$= \frac{1}{5}u^{5/2}+\frac{1}{3}u^{3/2}+C = \boxed{\frac{1}{5}(x^2-1)^{5/2}+\frac{1}{3}(x^2-1)^{3/2}+C}$$

3) a) $(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$ $f(1) = \sqrt{4} = 2 \Rightarrow f^{-1}(2) = 1$

$f'(x) = \frac{1}{2}\,\frac{3x^2+2x+1}{\sqrt{x^3+x^2+x+1}}$; $f'(1) = \frac{6}{4} = \frac{3}{2}$

$$= \frac{1}{f'(1)}$$

$$= \frac{1}{3/2} = \boxed{2/3}$$

b) Differentiate both sides with respect to x to get

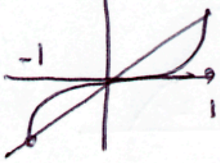
$$\sin\pi x + x\pi\cos\pi x = \frac{d}{dx}\int_0^{x^2} f(t)\,dt = f(x^2)\frac{d}{dx}x^2 = f(x^2)\cdot 2x$$

When $x=2$, $\sin 2\pi = 0$, $\cos 2\pi = 1$ so

$$2\pi\cdot 1 = f(4)\cdot 4$$

$$\Rightarrow f(4) = \boxed{\pi/2}$$

4)



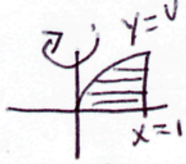
$x^3 = x \Rightarrow x = 0, \pm 1$

$$A = \int_{-1}^{1} |x - x^3|\,dx = 2\int_0^1 x - x^3\,dx \quad \text{(symmetry)}$$

$$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{4}\right] = \boxed{\frac{1}{2}}$$

$y = \sqrt{x}$, $x = y^2$

5)



$\rightarrow$

$$V = \int_0^1 [\pi(1)^2 - \pi(y^2)^2]\,dy = \int_0^1 (\pi - \pi y^4)\,dy = \pi\int_0^1 (1 - y^4)\,dy$$

$$= \pi\left[y - \frac{y^5}{5}\right]_0^1 = \pi\left[1 - \frac{1}{5}\right] = \boxed{\frac{4}{5}\pi}$$

6) Since $F(x)$, $G(x)$ are both anti-derivatives for $f(x)$

it follows that $F(x) = G(x) + C$

when $x = a$ $\quad 0 = \int_a^a f(t)\,dt = F(a) = G(a) + C \Rightarrow C = -G(a)$

Therefore $F(x) = G(x) - G(a)$

when $x = b$ $\quad \int_a^b f(t)\,dt = F(b) = G(b) - G(a)$

$$\therefore \int_a^b f(t)\,dt = G(b) - G(a)$$

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