Problem 1. (4 pts)

Let $A = \{1,2,3\}$ $B = \{2,4,6\}$

For each relation between A and B as a subset of $A \times B$, state whether it is a function from A to B, and if it is tell whether it is one to one or onto.

One point each part

a) $\{(1,4),(2,4),(3,6)\}$ A function, not 1-1 or onto

b) $\{(1,3),(2,6),(3,4)\}$ Not a function from A to B since $3 \notin B$

c) $\{(1,6),(1,2),(1,4)\}$ Not a function since 1 has multiple values

d) $\{(1,2),(4,3),(3,6)\}$ Not a function from A to B since $4 \notin A$

Problem 2. (4 pts) Show $Z^+ = \{1, 2, 3, \cdots\}$ and the set E of even positive integers have the same cardinality by giving a function $g : Z^+ \to E$ which is a one to one correspondence. Prove that g is a one to one correspondence.

2 pts

$g(n) = 2n$

1 pt

g 1-1 Since if $g(n) = g(m)$ then

$2n = 2m$

$\Rightarrow n = m$

1 pt

g onto Let $2k \in E$.

$g(n) = 2k$ if $2n = 2k$

$\Rightarrow n = k$

Problem 3. (4 pts) Let $f : Z \to Z$ be given by $f(n) = 2\lfloor \frac{n}{2} \rfloor$

Is f one to one? Explain

2 pts

No (1 pt)

$f(0) = 2 \lfloor \frac{0}{2} \rfloor = 2 \lfloor 0 \rfloor = 2 \cdot 0 = 0$

and $f(1) = 2 \lfloor \frac{1}{2} \rfloor = 2 \cdot 0 = 0$

} 1 pt

So $f(0) = f(1)$ but $0 \neq 1$

Is f onto? Explain.

2 pts

No (1 pt)

$f(n) = 2 \lfloor \frac{n}{2} \rfloor$ is always even, so no odd number is in the range of f } 1 pt

Problem 4. (4 pts)

a) Define f is $O(g)$ f is $O(g)$ if there are constants C, k such that

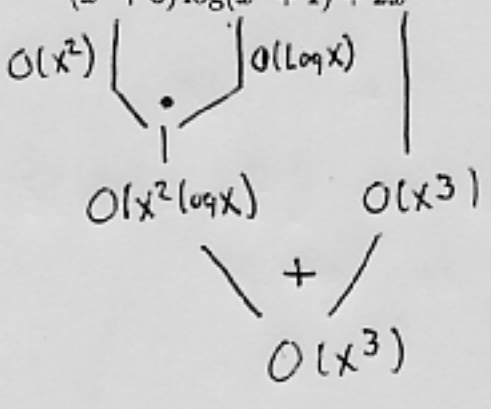
$|f(x)| \leq C|g(x)|$ whenever $x > k$

2 pts

b) For the function g in your estimate of f is $O(g)$ find a simple function of smallest order for the estimate of

2 pts

$(x^2+5)\log(x^4+1)+2x^3$



Since $\log x \leq x$ if $x > 0$

$x^2 \log x \leq x^3$ if $x > 0$

$O(x^3)$