

Problem 1. (4 pts)

Let $A = \{1, 2, 3\}$ $B = \{2, 4, 6\}$

For each relation between A and B as a subset of $A \times B$, state whether it is a function from A to B , and if it is tell whether it is one to one or onto.

- 1 pt
each
part
- a) $\{(2, 2), (1, 6), (3, 4)\}$ A function, both 1-1 and onto
- b) $\{(1, 2), (2, 6), (2, 4)\}$ Not a function, two values assigned to 2
- c) $\{(1, 6), (2, 2), (4, 6)\}$ Not a function from A to B
 $4 \notin A$
- d) $\{(1, 6), (2, 2), (3, 6)\}$ A function, not 1-1 or onto

Problem 2. (4 pts) Show $N = \{0, 1, 2, \dots\}$ and the set E of odd positive integers have the same cardinality by giving a function $g: N \rightarrow E$ which is a one to one correspondence. Prove that g is a one to one correspondence.

2 pts $g(n) = 2n+1$

1 pt g is 1-1 since

$$\begin{aligned}g(n) = g(m) &\Rightarrow 2n+1 = 2m+1 \\ &\Rightarrow 2n = 2m \\ &\Rightarrow n = m\end{aligned}$$

1 pt g is onto since given odd positive integer $2k+1$,

$$g(n) = 2k+1 \Rightarrow 2n+1 = 2k+1 \Rightarrow n = k$$

So $2k+1$ is in the range of g

Problem 3. (4 pts) Let $g: \mathbb{Z} \rightarrow \mathbb{Z}$ be given by $g(n) = \lfloor \frac{n}{2} \rfloor + 1$

Is g one to one? Explain

2 pts

No (1 pt)

$$g(0) = \lfloor \frac{0}{2} \rfloor + 1 = \lfloor 0 \rfloor + 1 = 0 + 1 = 1$$

$$g(1) = \lfloor \frac{1}{2} \rfloor + 1 = 0 + 1 = 1$$

So $g(0) = g(1)$ but $0 \neq 1$

} 1 pt

Is g onto? Explain.

2 pts

Yes, given any integer k , set (1 pt)

$$n = 2(k-1). \text{ Then } g(n) = \lfloor \frac{2(k-1)}{2} \rfloor + 1$$

$$= \lfloor k-1 \rfloor + 1$$

$$= k-1 + 1$$

$$= k$$

} (1 pt)

So g is onto

Problem 4. (4 pts)

a) Define f is $O(g)$ & g is $O(f)$ is there are constants C, k such that

2 pts

$$|f(x)| \leq C|g(x)| \text{ whenever } x > k$$

b) For the function g in your estimate of f is $O(g)$ find a simple function of smallest order for

$$2^n + (n^5 + 7n^2 + 1)(\log \frac{n}{4})^4$$

2 pts

$O(2^n)$

$O(n^5) \cdot O(n)$ { since $(\log n)^4 \leq n$ for $n > 1$

+ $O(n^5 \cdot n) = O(n^6)$

$O(2^n)$ since $n^6 < 2^n$ for large n

$$\boxed{O(2^n)}$$