Problem 1. (3 pts)

a) Convert $2AC3_{16}$ from hexadecimal to binary.

2 A C 3

0010 1010 1100 0011_2

b) Convert 347_{10} from decimal to octal (base 8).

$\div 8 \quad 347 = 8 \cdot 43 + 3$

$\div 8 \quad 43 = 8 \cdot 5 + 3$

$\div 8 \quad 5 = 8 \cdot 0 + 5$

$\boxed{533_8}$

c) Convert 101101_2 from binary to decimal.

1

0 $\quad 2 \cdot 1 + 0 = 2$

1 $\quad 2 \times 2 + 1 = 5$

1 $\quad 2 \times 5 + 1 = 11$

0 $\quad 2 \times 11 + 0 = 22$

1 $\quad 2 \times 22 + 1 = \boxed{45_{10}}$

Problem 2. (4 pts)

Let $a_0 = 1; a_1 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$ for $n \geq 3$.

a) Find a_4

1 pt

$a_2 = 2a_1 + a_0 = 2 \cdot 1 + 1 = 3$

$a_3 = 2 \cdot a_2 + a_1 = 2 \cdot 3 + 1 = 7$

$a_4 = 2 \cdot a_3 + a_2 = 2 \cdot 7 + 3 = \boxed{17}$

b) Use Mathematical Induction to prove that a_n is odd for all $n \geq 0$.

1pt

Basis Step $a_0 = 1$ is odd ✓

2 pts

Induction Step Assume a_k odd for $0 \leq k \leq n$

Consider $a_{n+1} = 2a_n + a_{n-1}$

induction hypothesis $\Rightarrow \begin{cases} a_n \text{ odd} \Rightarrow 2a_n \text{ even} \\ a_{n-1} \text{ odd} \end{cases}$

Since even+odd is odd

a_{n+1} is odd $(a_{n+1} = \underbrace{2a_n}_{\text{even}} + \underbrace{a_{n-1}}_{\text{odd}} - \text{odd})$

Therefore by 2^{nd} PMI a_n odd for all $n \geq 0$

Problem 3. (3 pts)

Find a recurrence relation for the number of ternary strings of length $n \geq 1$ that contain an even number of 1's. Note that 0 is an even number. Also give the initial conditions. (A ternary string is a string consisting of the digits 0,1,2.)

2pts

$$a_n \begin{cases} 0\ \boxed{a_{n-1}} \\ 2\ \boxed{a_{n-1}} \\ 1 - \underbrace{\boxed{\text{odd number of 1's}}}_{3^{n-1} - a_{n-1}} \end{cases}$$

$$a_n = 2a_{n-1} + 3^{n-1} - a_{n-1}$$

$$\boxed{a_n = a_{n-1} + 3^{n-1}}$$

1pt Initial conditions

$a_0 = 1$ (empty string contains zero 1's which is even)

or

$a_1 = 2$ (strings 0, 2 contain even number of 1's)

Problem 4. (3 pts)

a) How many functions are there from a set with 5 elements to a set with 6 elements?

$\underline{6^5}$

b) How many are one to one?

$\underline{P(6,5)} = \frac{6!}{(6-5)!} = \underline{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}$

c) How many are onto?

None, since Codomain is larger than domain

Problem 5.(3 pts)

a) How many bit strings are there of length 10 which have two or more 0's.

Total number of bit strings 2^{10}

number with no zeros 1

number with one zero 10

$$2^{10} - 1 - 10 = \boxed{2^{10} - 11}$$

b) How many students must be in a class to guarantee that at least 5 were born on the same day of the week? Find the least number.

$$\left\lceil \frac{N}{7} \right\rceil = 5 \qquad N = 7 \cdot 4 + 1 = \boxed{29}$$

c) How many ways are there to choose a bag of 12 cookies from 5 varieties, including chocolate chip, if at least four chocolate chip cookies must be chosen.

4 must be chocolate chip, so only 8 choices

$n =$ number of types $= 5$

$k =$ number choosen $= 8$

$$C(n+k-1, k) = C(5+8-1, 8) = \boxed{C(12, 8)}$$