Problem 1. (3 pts)

a) Convert $A2B3_{16}$ from hexadecimal to binary.

$1010\ 0010\ 1011\ 0011_2$

b) Convert 425_{10} from decimal to octal (base 8).

$\div 8 \quad 425 = 8 \cdot 53 + 1$

$\div 8 \quad 53 = 8 \cdot 6 + 5$

$\div 8 \quad 6 = 8 \cdot 0 + 6$

$\boxed{651_8}$

c) Convert 110101_2 from binary to decimal.

1

1 $\quad 2 \times 1 + 1 = 3$

0 $\quad 2 \times 3 + 0 = 6$

1 $\quad 2 \times 6 + 1 = 13$

0 $\quad 2 \times 13 + 0 = 26$

1 $\quad 2 \times 26 + 1 = \boxed{53_{10}}$

Problem 2. (4 pts)

Let $c_0 = 0; c_1 = 2; c_2 = 2$ and $c_n = c_{n-1} + 3c_{n-2}$ for $n \geq 3$.

1pts a) Find c_4

$C_3 = C_2 + 3C_1 = 2 + 3\cdot 2 = 2+6 = 8$

$C_4 = C_3 + 3\cdot C_2 = 8 + 3\cdot 2 = 8+16 = \boxed{14}$

b) Use Mathematical Induction to prove that c_n is even for all $n \geq 0$.

1pt Base Step $C_0 = 0$ even ✓

2pts Induction Step Assume C_k even for $0 \leq k \leq n$

Consider $C_{n+1} = C_{n-1} + 3C_{n-2}$

Induction hypothesis $\Rightarrow$ C_{n-1} even

C_{n-2} even $\Rightarrow 3C_{n-2}$ even

even + even = even

Therefore

$C_{n+1} = \overset{\text{even}}{C_{n-1}} + \overset{\text{even}}{3C_{n-2}}$ is even

Hence, by PMI C_n even for all $n \geq 0$

Problem 3. (3 pts)

Find a recurrence relation for the number of ternary strings of length $n \geq 1$ that contain an odd number of 2's. Note that 0 is an even number. Also give the initial conditions. (A ternary string is a string consisting of the digits 0,1,2.)

2pts

$$a_n = \left\langle \begin{array}{l} 1\ \boxed{a_{n-1}} \\ 0\ \boxed{a_{n-1}} \\ 2\ \underbrace{\boxed{\text{even number of 2's}}}_{3^{n-1}-a_{n-1}} \end{array} \right.$$

$$a_n = 2a_{n-1} + 3^{n-1} - a_{n-1}$$

$$= \underline{a_{n-1} + 3^{n-1}}$$

1pt initial conditions

$a_0 = 0$ (Since empty string contains an even numbe of 2's (zero 2's)

or

$a_1 = 1$ (The string 2)

Problem 4. (3 pts)

a) How many functions are there from a set with 5 elements to a set with 7 elements?

$\underline{7^5}$

b) How many are onto?

<u>none</u> since codamain larger than domain

c) How many are one to one?

$\underline{P(7,5)} = \frac{7!}{(7-5)!} = \underline{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}$

Problem 5.(3 pts)

a) How many bit strings are there of length 12 which have two or more 1's.

Total number 2^{12}

no 1's 1

one 1 12

$2^{12}-1-12 = \underline{2^{12}-13}$

b) How many students must be in a class to guarantee that at least 6 were born on the month?

$\left\lceil \frac{N}{12} \right\rceil = 6 \qquad N = 12 \cdot 5 + 1$

$= \underline{61}$

c) How many ways are there to choose a bag of 15 cookies from 6 varieties, including chocolate chip, if at least five chocolate chip cookies must be chosen.

only $15-5 = 10$ choices

$n =$ no of types $= 6$

$k =$ no of choices 10

$C(n+k-1, k) = C(6+10-1, 10) = \boxed{C(15,10)}$