Prob Set Feb 27

5.3) 4) $3^{47} = 3^{2\cdot 22+3} = \left(3^{22}\right)^2 \cdot 3^3 \equiv 1^2\cdot 3^3 \equiv 27 \equiv 4 \mod 23$

14) $45x \equiv 15 \mod 24$ Since $45 \equiv 21 \mod 24$

$\Leftrightarrow 21x \equiv 15 \mod 24$ $\gcd(21,24)=3$ and $3 \mid 15$ so $\exists$ solution to reduced equation

$7x \equiv 5 \mod 8$ $1 = 8 + (-1)\cdot 7$ so -1 inverse of $7 \mod 8$

$x \equiv (-1)7x \equiv -5 \mod 8$

$x \equiv 3 \mod 8$ $\left\{\text{Solutions}\right.$ $3 + 24\cdot n$, $11 + 24\cdot n$, $19 + 24\cdot n$ $n \in \mathbb{Z}$

5.4) 8) $[(-a,b)] + [(a,b)] = [(-ab+ba, b^2)]$

$= [(0,b^2)] = [(0,1)]$ since $(0,b^2) \sim (0,1)$

additive identity

5.6 10) $-1\ (=6)$ is a zero, so $x+1$ is a factor

$$\begin{array}{r|l} & x^2 + x + 1 \\ x+1 & x^3 + 2x^2 + 2x + 1 \\ & \underline{x^3 + x^2} \\ & \quad x^2 + 2x \\ & \quad \underline{x^2 + x} \\ & \qquad x + 1 \end{array}$$

2 is a zero of x^2+x+1 so $x-2$ is a factor

$$\begin{array}{r|l} & x + 3 \\ x-2 & x^2 + x + 1 \\ & \underline{x^2 - 2x} \\ & \quad 3x + 1 \\ & \quad \underline{3x - 6} \\ & \qquad 7 = 0 \end{array}$$

Thus

$x^3 + 2x^2 + 2x + 1 = (x+1)(x-2)(x+3)$

$= (x-6)(x-2)(x-4)$ in $\mathbb{Z}_7$

32) if $p \nmid a$ $a^p \equiv a \mod p$

if p odd prime $(-a)^p = (-1)^p a^p \equiv -a \mod p$ so $x = -a$ zero of $x^p + a$ in $\mathbb{Z}_p[x]$

and $x^p + a$ reducible

if $p = 2$, $a = 0, 1$ and

both $x^2 + 0 = x\cdot x$ and $x^2 + 1 = (x+1)(x+1)$ are reducible in $\mathbb{Z}_2[x]$