March 11 Prob Set

5.6) 12) -1 is a zero

$$\begin{array}{r} x^2 - x + 3 \\ x+1 \overline{) x^3 \quad + 2x + 3} \\ \underline{x^3 + x^2} \\ -x^2 + 2x \\ \underline{-x^2 - x} \\ 3x + 3 \\ \underline{3x + 3} \\ 0 \end{array}$$

-1 zero of $x^2 - x + 3$

$$\begin{array}{r} x - 2 \\ x+1 \overline{) x^2 - x + 3} \\ \underline{x^2 + x} \\ -2x + 3 \\ \underline{-2x - 2} \\ 5 = 0 \end{array}$$

$\therefore$

$x^3 + 2x + 3 = (x+1)^2(x-2)$

$= (x-4)^2(x-2)$ in Z_5

28) Listing all polynomials of degree 3

x^3	no, 0 is a zero	$\underline{x^3+x+1 \text{ irreducible}}$
x^3+1	no 1 is a zero	$\underline{x^3+x^2+1 \text{ irreducible}}$
x^3+x	no 0 is a zero	x^3+x^2+x no, 0 is a zero
x^3+x^2	no 0 is a zero	x^3+x^2+x+1 no, 1 is a zero

6.2 6) Need to find c such that x^3+x^2+c irreducible in $Z_3[x]$

Let $f(x) = x^3+x^2$ $\quad f(0)=0$, $f(1)=2$, $f(2)=0$

$x^3+x^2+c = f(x)+c$

So need $0+c,\ 2+c \neq 0$

only choice is $\underline{c=2}$

20) If N prime ideal R/N is a finite integral domain, hence a field. Therefore N is a maximal ideal

8.1) Here are some sample calculations in $Z_3(\alpha)$

$(1+\alpha)+(2+\alpha) = 3+2\alpha = 2\alpha$

$(1+\alpha)(2+\alpha) = 2+3\alpha+\alpha^2 = 2+\alpha^2$ But $\alpha^2+1=0$ so $\alpha^2=-1=2$

$= 2+2$

$= 1$