

Problem 1.

- a) Show that if  $G_1$  and  $G_2$  are subgroups of a group  $G$  then so is  $G_1 \cap G_2$ .
- b) Show that if  $H$  is a subgroup of group  $G$ , and  $g \in G$  then the set  $gHg^{-1} = \{ghg^{-1} \mid h \in H\}$  is also a subgroup of  $G$ .

Problem 2.

Find all the subgroups of  $Z_{21}$  and draw the lattice diagram for the subgroups.

Problem 3.

Find the product of the following permutations in  $S_8$

- a)  $(1,3)(6,3,2,8)$
- b)  $(7,8,2,1)(4,5)(5,6,4)$
- c) Write the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 3 & 2 & 5 & 1 & 6 & 7 & 4 \end{pmatrix}$$

as product of disjoint cycles, and then as a product of transpositions.

Problem 4.

Determine if the following are subgroups of the indicated groups. If not, say why not.

- 1)  $\{1, -1\} \subseteq \langle R, \times \rangle$
- 2) The set of all  $n$  by  $n$  matrices with determinant  $-1$  or  $1$  in  $GL_n(R)$ .
- 3) The set of positive integers in  $\langle Z, + \rangle$
- 4) The set of all  $n$  by  $n$  matrices with determinant  $-1$  in  $GL_n(R)$
- 5) The set of two cycles in  $S_n$

Problem 5.

Mark true or false.

- 1) Every element of a cyclic group generates the group.
- 2) The group  $S_3$  is cyclic.
- 3) Every cyclic group is abelian.
- 4) Every finite group with at most three elements is abelian.
- 5) Every element of a cyclic group generates the group.

Problem 6.

Let  $G$  be a group and  $Aut(G)$  be the set of all isomorphisms from  $G$  onto  $G$ . Prove that  $Aut(G)$  is a group under composition of functions.

Problem 7.

Let  $\phi : G \rightarrow H$  be a function from group  $G$  into group  $H$  which satisfies  $\phi(g_1g_2) = \phi(g_1)\phi(g_2)$ .

a) Show  $\phi(e) = e'$  where  $e$  is the identity of  $G$  and  $e'$  is the identity of  $H$ , and  $\phi(x^{-1}) = \phi(x)^{-1}$

b) Show that the set  $K = \{g \in G \mid \phi(g) = e'\}$  is a subgroup of  $G$