# ANOVA

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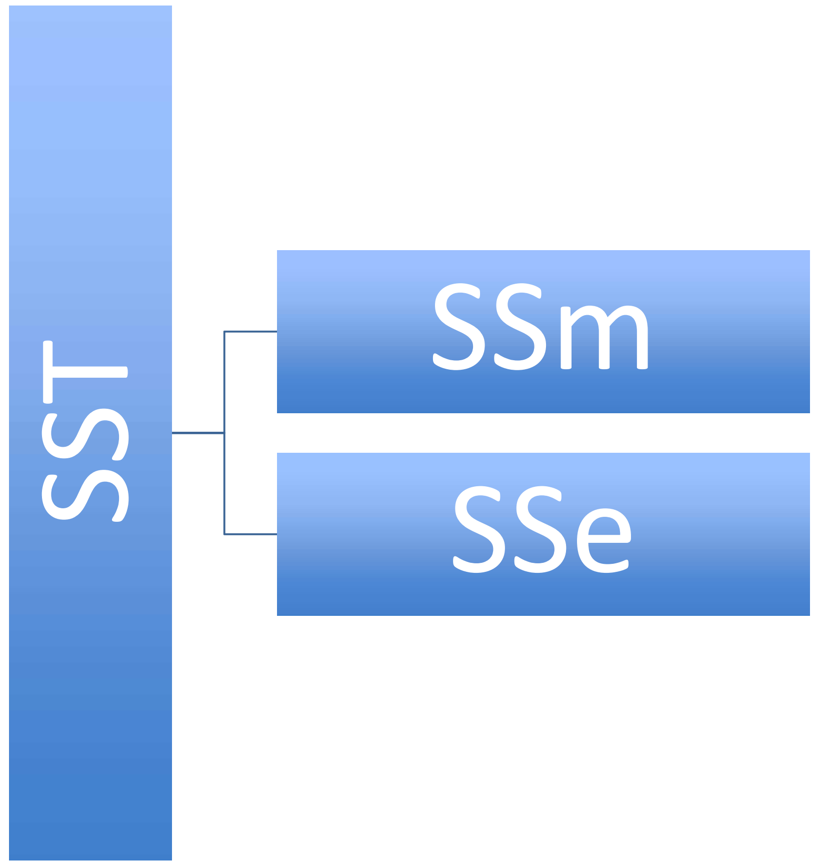
# ANOVA

- One-way independent ANOVA
- Two-way factorial ANOVA
- One-way repeated-measures
- Other ANOVAs
  - Two-way repeated
  - Two-way mixed
  - 3-way independent
  - Nested

# Key Components

- Effect
  - Variance explained by the predictor/model
- Error
  - Unexplained variance
- Assumptions
  - Relative distribution normality
  - Homogeneity of variance
- Procedural examples
  - Exploring the data (graphs; testing for relative normality)
  - Omnibus tests (ANOVA)
  - Pairwise tests controlling for family wise error
  - Effect size

# One-way ANOVA



# One-way ANOVA

- Effect (SSm) and error (SSe)
  - Total variance: SST= $\Sigma X^2 - (G)^2/N$
  - Effect (SSm aka SSbtw): $\Sigma(T)^2/n - (G)^2/N$
    - Variability in DV accounted for by a treatment
    - Differences in means among the treatments
  - Error (SSe aks SSw): SSe= $\Sigma SS$
    - Contribution of individual subject differences
    - Unsystematic, non-treatment related

$$F = \frac{\text{systematic treatment effect + unsystematic differences}}{\text{Unsystematic differences}}$$

# Simulated data

| Read 4 Speed | Read Well | Eye Candy |
|---|---|---|
| 8.24993 | 10.466106 | 8.949562 |
| 3.851183 | 6.934875 | 9.539441 |
| 5.166481 | 5.192835 | 8.415509 |
| 5.819707 | 11.630756 | 9.850214 |
| 8.554169 | 9.662392 | 9.303608 |
| 2.5191 | 7.690234 | 10.49997 |
| 4.331016 | 7.267778 | 10.221738 |

# Two-way

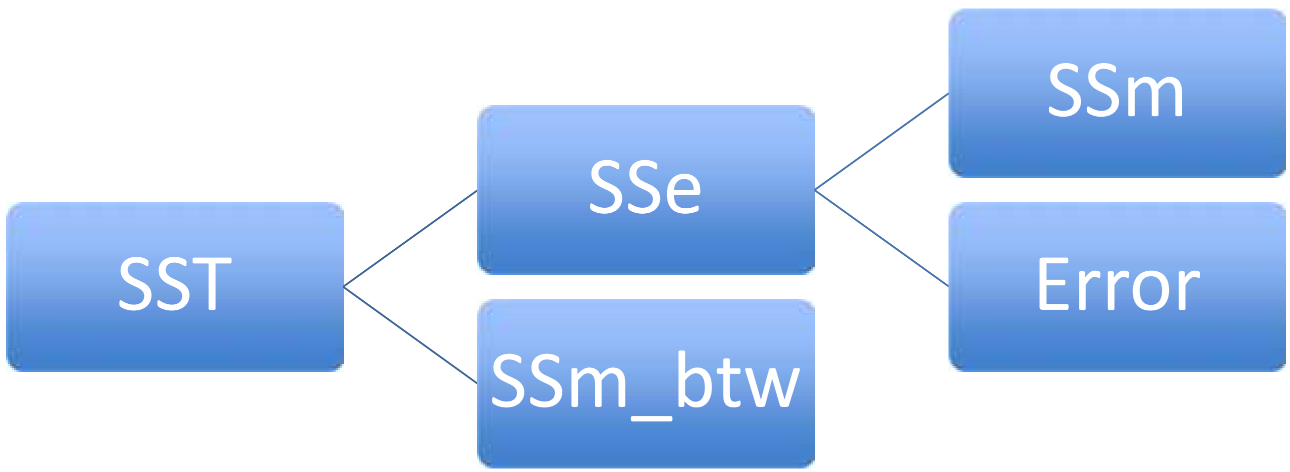
| gender | status | Anxiety_leve |
|---|---|---|
| female | lecture | 5 |
| female | lecture | 4 |
| female | lecture | 11 |
| female | lecture | 13 |
| female | exam | 14 |
| female | exam | 19 |
| female | exam | 11 |
| female | exam | 8 |
| male | lecture | 16 |
| male | lecture | 5 |
| male | lecture | 3 |
| male | lecture | 7 |
| male | exam | 20 |
| male | exam | 12 |
| male | exam | 13 |
| male | exam | 17 |

# Two-way

- Stage 1
  - Total variance: $SST = \Sigma X^2 - (G)^2/N$
  - Effect (SSm aka SSbtw): $\Sigma(T)^2/n - (G)^2/N$
  - Error (SSe aks SSw): $SSe = \Sigma SS$
- Stage 2
  - $SS_A = \Sigma(T)^2/n\_row - (G)^2/N$
  - $SS_B = \Sigma(T)^2/n\_col - (G)^2/N$
  - $SS_{AXB} = SSm - SS_A - SS_B$

| | | Factor B | BDI_Anxiety |
|---|---|---|---|
| | | Lectures | Exams |
| | | 5 | 14 |
| Factor A | Female | 4 | 19 |
| Gender | | 11 | 11 |
| | | 13 | 8 |
| | | 16 | 20 |
| | Male | 5 | 12 |
| | | 3 | 13 |
| | | 7 | 17 |

# Repeated-measures ANOVA



# Repeated measures

| subject | Read 4 Speed | Read Well | Eye Candy |
|---|---|---|---|
| s1 | 8.24993 | 10.466106 | 8.949562 |
| s2 | 3.851183 | 6.934875 | 9.539441 |
| s3 | 5.166481 | 5.192835 | 8.415509 |
| s4 | 5.819707 | 11.630756 | 9.850214 |
| s5 | 8.554169 | 9.662392 | 9.303608 |
| s5 | 2.5191 | 7.690234 | 10.49997 |
| s6 | 4.331016 | 7.267778 | 10.221738 |

# Repeated-measures ANOVA

- Stage 1
  - Total variance: SST= $\Sigma X^2 - (G)^2/N$
  - Effect (SSm aka SSbtw): $\Sigma(T)^2/n - (G)^2/N$
  - Error (SSe aka SSw): SSe= ΣSS
- Stage 2
  - SS_per= $\Sigma(P)^2/k - (G)^2/N$
    - *Individual differences removed from denominator*
    - SSerror = SSe- SS_per

*F* = systematic treatment effect, unsys. diffs (excluding individual differences)
Unsystematic random differences (excluding individual differences)

# Assumptions

- ANOVA one-way
  - Homogeneity of variance
    - Variances across treatments should be similar
    - Levene Test
    - Dependent variable: interval or ratio scale
  - Relatively normal distribution
  - Residuals should have a relatively normal distribution
  - Non-normal data: non-parametric or robust tests
    - Kruskal-Wallis; T1way (Wilcox, 2005)

# Assumptions

- Two-way ANOVA
  - Same as those in one-way
  - Plus complication
    - Variance can come from two factors
    - Homogeneity of variance
      - Assessed for main effects
      - As well as the Interaction

# Assumptions

- Repeated measures ANOVA
  - Assumption of sphericity
    - Similar to homogeneity of variance
    - Assumes equality of variances on scores between treatment levels

# Example procedures

- Power test (optional)
  - Important to know *n* required
    - to achieve a large effect size
    - at least 80% power
- Explore the data
  - Shapiro-Wilks test
  - Levene Test
  - Plot residuals
- Omnibus test
  - Pairwise test
  - Effect size

# Simulated data

- Simulalated data
  - 3 different reading compression programs
  - The dependent is  a score (in minutes)
    - composite of correct answer and RT
- The independent variable
  - Reading method
- Call in R
  - R4S<-runif(7, min= 2, max=10)
  - RW<-runif(7, min= 5, max=13)
  - EC<-runif(7, min=6, max=12)

# Simulated data

| Read 4 Speed | Read Well | Eye Candy |
|---|---|---|
| 8.24993 | 10.466106 | 8.949562 |
| 3.851183 | 6.934875 | 9.539441 |
| 5.166481 | 5.192835 | 8.415509 |
| 5.819707 | 11.630756 | 9.850214 |
| 8.554169 | 9.662392 | 9.303608 |
| 2.5191 | 7.690234 | 10.49997 |
| 4.331016 | 7.267778 | 10.221738 |

# Explore the data

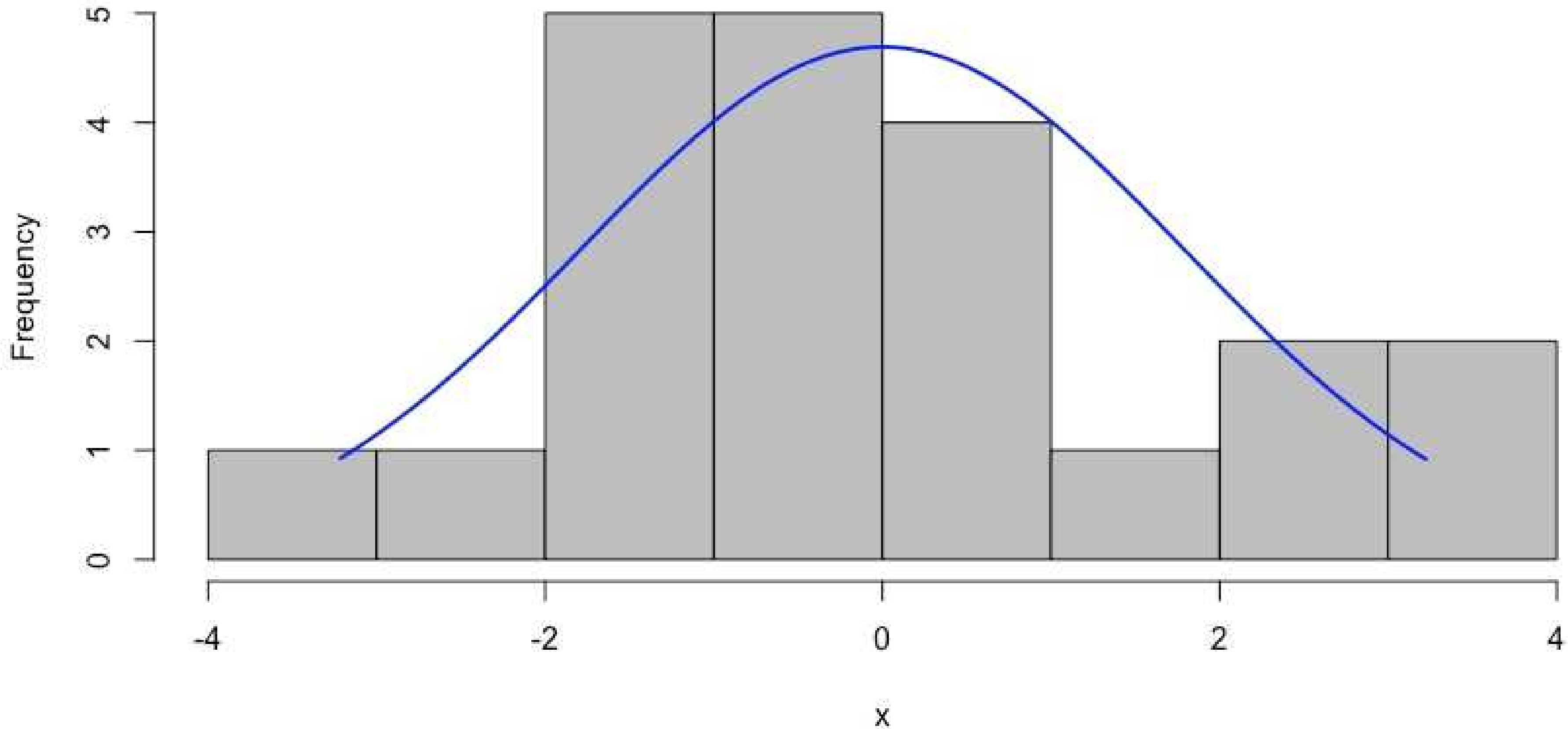
- Shapiro-Wilk test of normality
  - Biased for larger samples
  - May be non-normal but hidden by *larger N*
- Current data; shapiro.test(score)
  - data:  score
  - W = 0.9474, p-value = 0.3039

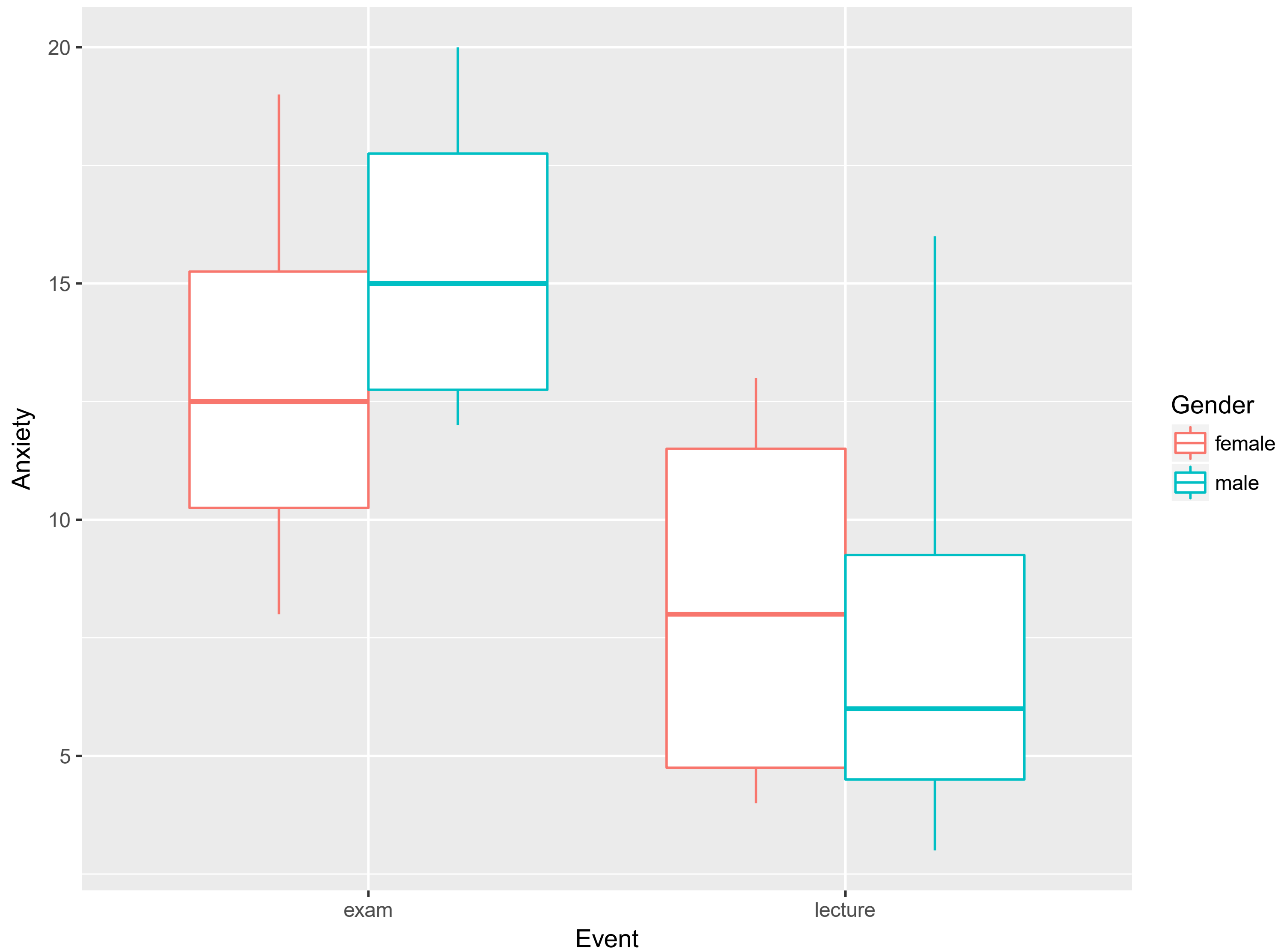
# Explore the data

- Test homogeneity of variance

  Levene's Test for Homogeneity of Variance (center = median)

| | Df | F value | Pr(>F) |
|---|---|---|---|
| group | 2 | 2.4518 | 0.1144 |
| | 18 | | |


Frequency
0
1
2
3
4
5
-4
-2
0
2
4
x



20
15
10
5
Anxiety
exam
lecture
Event
Gender
female
male

# One-way

- tapply(dat1$score, dat1$method, var)
  - Read 4 Speed    Read Well    Eye Candy

  5.0123978    5.0863527    0.5252534
- tapply(dat1$score, dat1$method, mean)
  - Read 4 Speed    Read Well    Eye Candy

  5.498798    8.406425    9.540006
- tapply(dat1$score, dat1$method, sd)
  - Read 4 Speed    Read Well    Eye Candy

  2.2388385    2.2552944    0.7247437

# One-way results

- mod1<-aov(score~method, data= dat1)
  - summary(mod1)

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) | |
|---|---|---|---|---|---|---|
| method | 2 | 60.83 | 30.416 | 8.589 | 0.0024 | ** |
| Residuals | 18 | 63.74 | 3.541 | | | |

- eta-squared of multiple R^2
  - summary.lm(mod1)
  - Multiple R^2= .48, referred to as eta-squared in ANOVA (SSbtn/SST)

# Pairwise test

- pairwise.t.test(dat1$score, dat1$method, p.adjust.method = "bonf" )

data:  dat1$score and dat1$method

| | Read 4 Speed | Read Well |
|---|---|---|
| Read Well | 0.0292 | - |
| Eye Candy | 0.0024 | 0.8237 |

# Effect size

- Read 4 Speed vs. Read Well
  - mes(5.498798 , 8.406425, 2.2388385 , 2.2552944, 7, 7)
  - $r^2$= .37
- Read 4 Speed vs Eye Candy
  - mes(5.498798, 9.540006 , 2.2388385, 0.7247437, 7, 7)
  - $r^2$= .59

# Factorial two-way

- Shapiro-Wilk test of normality
  - Biased for larger samples
  - May be non-normal but hidden by *larger N*
- Current data;
- Shapiro-Wilk normality test
  - data: two_way$Anxiety
  - W = 0.95404, p-value = 0.5563

# Factorial two-way

- Test homogeneity of variance
  - Event (Exam or Lecture)

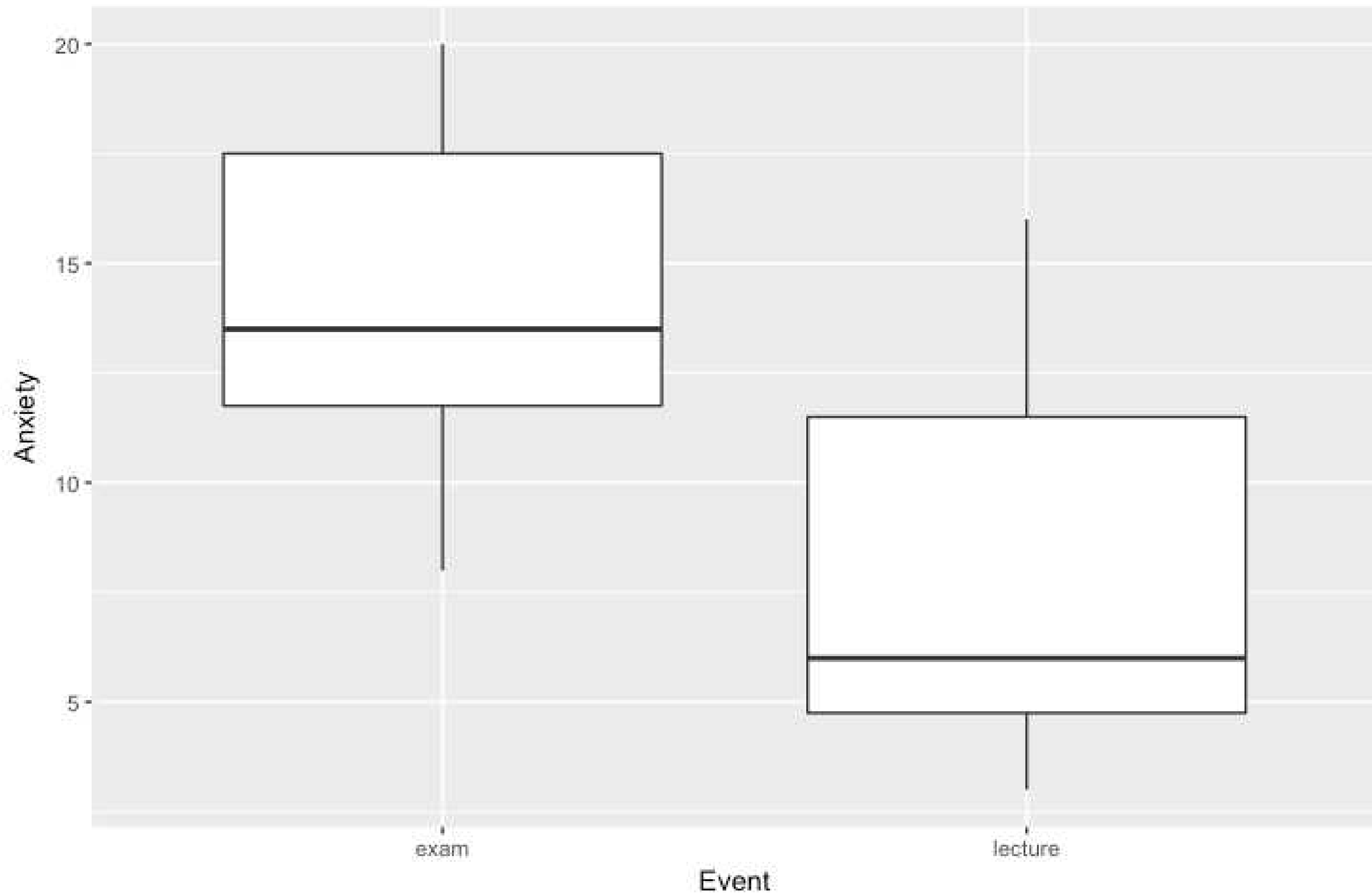
- Levene's Test for Homogeneity of Variance (center = median)

| | Df | F value | Pr(>F) |
|---|---|---|---|
| group | 1 | 0.1197 | 0.7345 |
| | 14 | | |

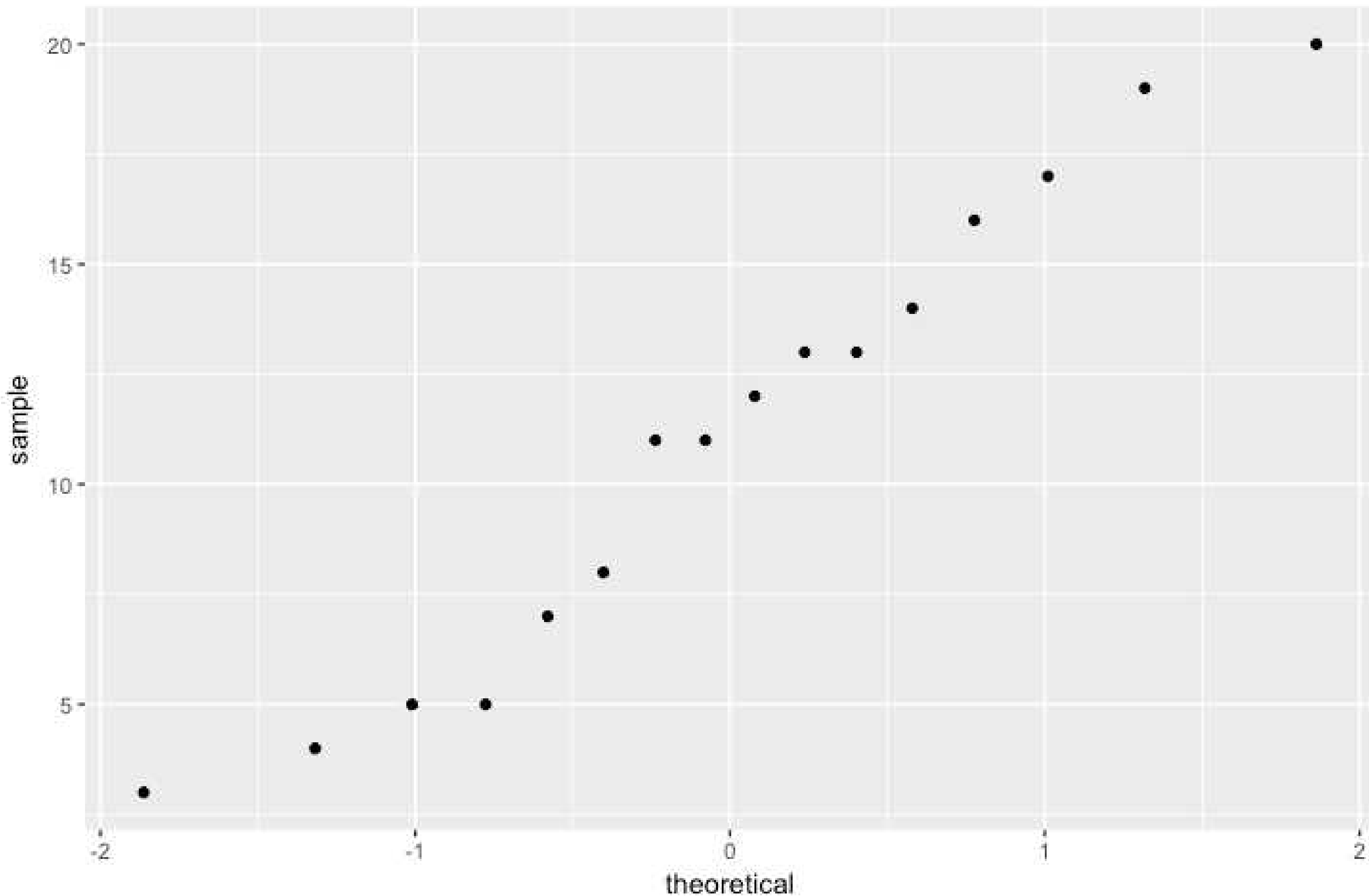
# Factorial two-way

- Testing for violation of
  - Homogeneity of variance from interaction
  - leveneTest(two_way$Anxiety, interaction(two_way$Gender, two_way$Event))

```
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  3  0.0736  0.973
      12
```



20
15
10
5
Anxiety
exam
lecture
Event



20
15
10
5
sample
-2
-1
0
1
2
theoretical

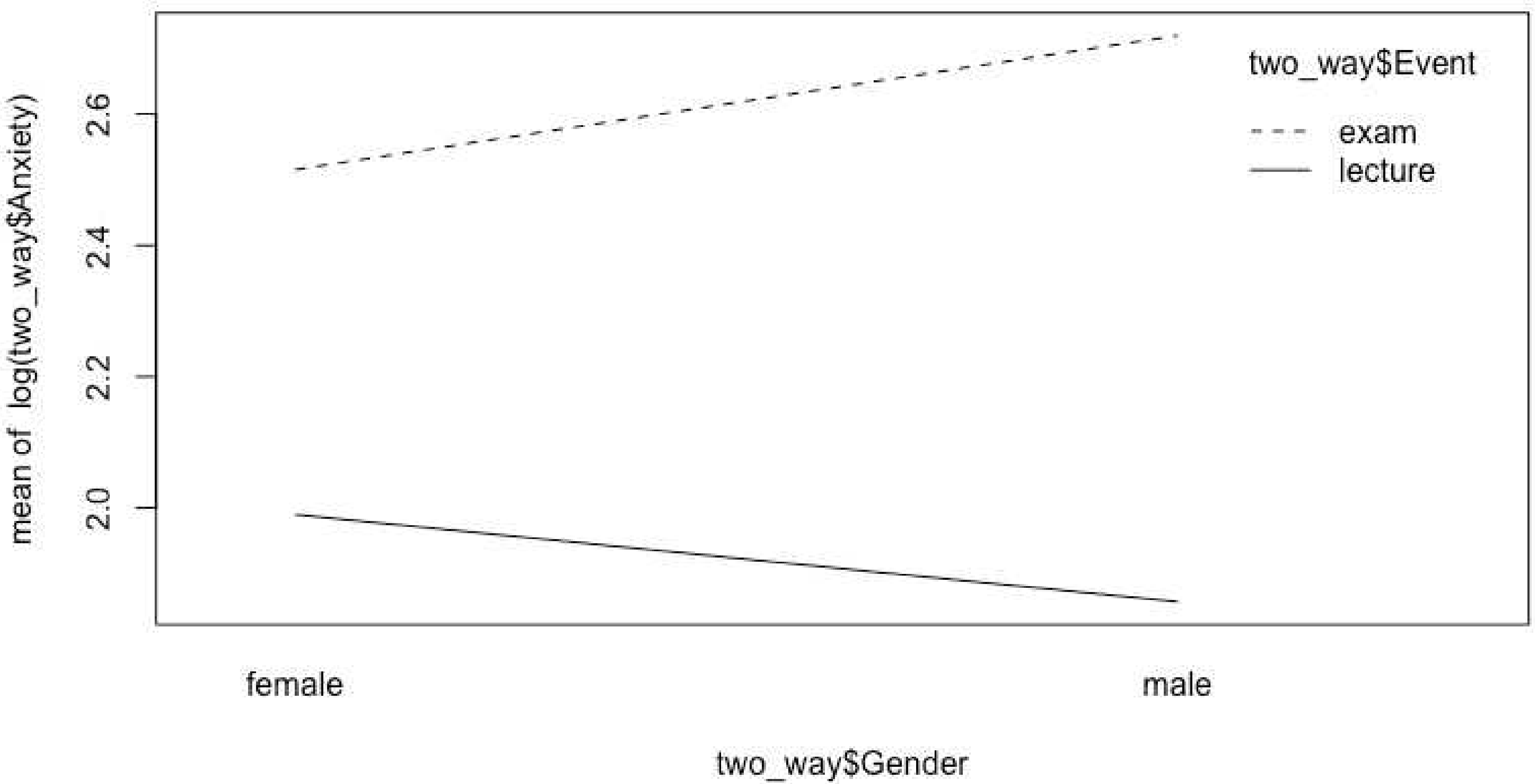
# Two-way

- tapply(two_way$Anxiety, two_way$Event, mean)
  - shows mean anxiety based on Event
  - Exam lecture
    - 14.25 8.00
- tapply(two_way$Anxiety, two_way$Gender, mean)
  - shows the mean anxiety based on Gender
  - female male
    - 10.625 11.625

# Two-way factorial

- mod3<-aov(Anxiety~Event + Gender + Event:Gender, data = two_way)
  - summary(mod3)

| | Df | Sum Sq | Mean Sq | F | value Pr(>F) |
|---|---|---|---|---|---|
| # Event | 1 | 156.2 | 156.25 | 7.089 | 0.0207 * |
| # Gender | 1 | 4.0 | 4.00 | 0.181 | 0.6777 |
| # Event:Gender | 1 | 9.0 | 9.00 | 0.408 | 0.5348 |
| # Residuals | 12 | 264.5 | 22.04 | | |



two_way$Event
exam
lecture
mean of log(two_way$Anxiety)
2.0
2.2
2.4
2.6
female
male
two_way$Gender

# Repeated measures

| subject | Read 4 Speed | Read Well | Eye Candy |
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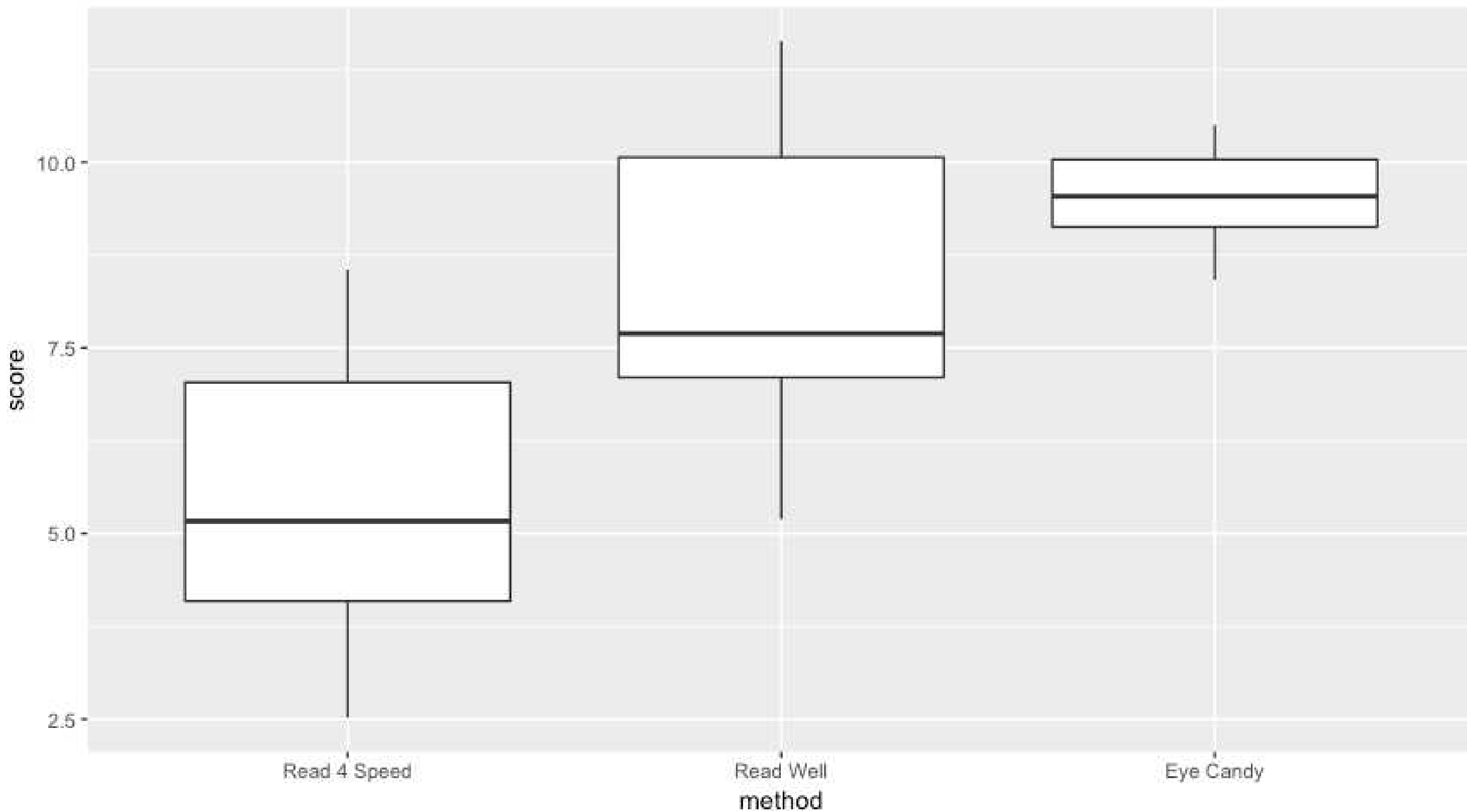
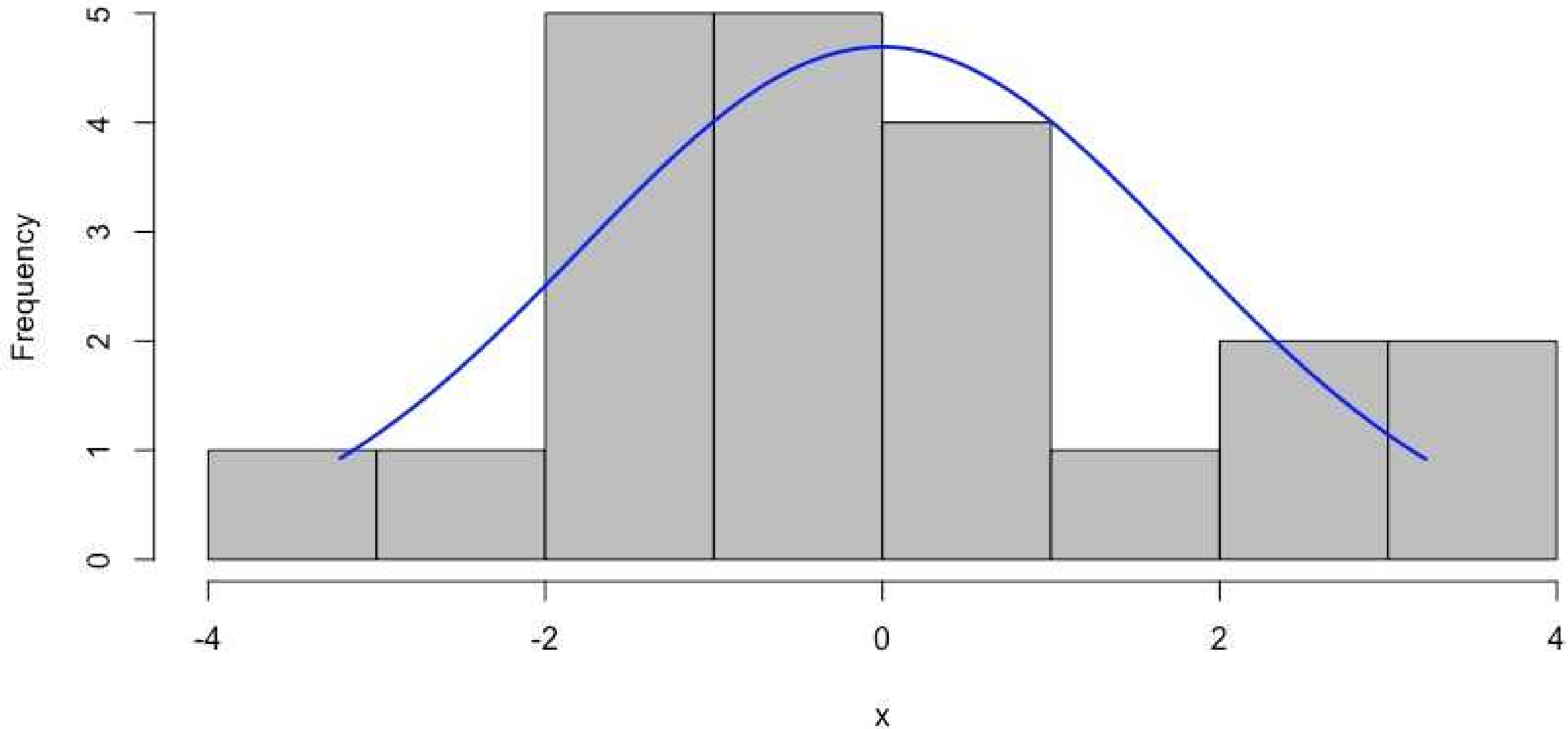


10.0
7.5
5.0
2.5
score
Read 4 Speed
Read Well
Eye Candy
method



Frequency
0
1
2
3
4
5
-4
-2
0
2
4
x

# One-way

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  - Read 4 Speed Read Well Eye Candy

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  - Read 4 Speed Read Well Eye Candy

  5.498798 8.406425 9.540006
- tapply(dat1$score, dat1$method, sd)
  - Read 4 Speed Read Well Eye Candy

  2.2388385 2.2552944 0.7247437

# Repeated measures ANOVA

- Call
  - mod5<-ezANOVA(data=rep_m, dv=.(score), wid= .(subject), within= .(method), detailed= TRUE, type=2)

```
$ANOVA
  Effect  DFn DFd    SSn         SSd     F       p     p<.05        ges
method   2   12  60.83155  33.42687 10.91904 1.989086e-03   * 0.4883104

$`Mauchly's Test for Sphericity`
 Effect    W            p p<.05
2 method 0.887657     0.7423572

$`Sphericity Corrections`
 Effect    GGe    p[GG] p[GG]<.05    HFe    p[HF] p[HF]<.05
2 method 0.8990033 0.003023708     * 1.259646 0.001989086     *
```

- Note that this is a higher F-ratio value (10.91) relative to the Independent one-way (8.59)
    - Recall, error in the denominator is reduced
        - Individual differences are removred

# Repeated measures multilevel

- Sphericity is not a concern
- Form of regression
  - Explicitly models dependency
    - (Normally residuals must be independent in regression)
  - Includes fixed coefficients and random coefficents
    - Fixed: e.g. gender
    - Random: something that changes over time
  - Fixed effects generalize
    - only to the sample and experiment
  - Random effects generalize beyond the experiment (Field, 2013)

# Repeated measures multilevel

- baseline<-lme(score~1, random= ~1|subject, data= rep_m, method= "ML”)
  - mod6<-lme(score~method, random= ~1|subject, data=rep_m, method="ML")

- anova(baseline, mod6)

# Repeated measures multilevel

| | Model | df | AIC | BIC | logLik | Test | L.Ratio | p-value |
|---|---|---|---|---|---|---|---|---|
| baseline | 1 | 3 | 102. | 106. | -48 | | | |
| mod6 | 2 | 5 | 92. | 97. | -41.1 | vs 2 | 14.94333 | 6e-04 |

- The AIC and BIC value measures how well a model fits the data: lower is better
- The L.Ratio:
  - indicates the model fits the data significantly better than no model (baseline

# Nested ANOVA

| Tech: | Patel | | | Jenkins | | |
|---|---|---|---|---|---|---|
| Rat: | Alice | Freda | Willy | Mary | Patch | Amy |
| | 0.3 | 0.09 | 1.27 | 0.87 | 0.4829 | 1.376662 |
| | 0.5 | 0.15 | 1.45 | 1.05 | 0.6035 | 1.47073 |
| | 0.7 | 0.21 | 1.63 | 1.23 | 0.7241 | 1.564798 |
| | 0.9 | 0.27 | 1.81 | 1.41 | 0.8447 | 1.658866 |
| | 1.1 | 0.33 | 1.99 | 1.59 | 0.9653 | 1.752934 |
| | 1.3 | 0.39 | 2.17 | 1.77 | 1.0859 | 1.847002 |
| | 1.5 | 0.45 | 2.35 | 1.95 | 1.2065 | 1.94107 |
| | 1.7 | 0.51 | 2.53 | 2.13 | 1.3271 | 2.035138 |
| | 1.9 | 0.57 | 2.71 | 2.31 | 1.4477 | 2.129206 |