Research Statement

Algebraic combinatorics is my research area, with the focus on Jones pairs, type II matrices and their relations to association schemes. Association schemes encode combinatorial objects such as distance regular graphs and symmetric designs. They can also be viewed as a generalization of finite abelian groups. Two families of association schemes, the Johnson schemes and the Hamming schemes, have applications in the study of designs, codes and orthogonal arrays. My interest lies in the connections of association schemes with coding theory, design theory, graph theory and knot theory.

In 1989, Jones introduced spin models to construct link invariants. Spin models can be viewed as matrices that satisfy three types of conditions corresponding to the three Reidemeister moves on link diagrams [5]. In particular, an $n \times n$ matrix $W$ satisfies the condition corresponding to the second Reidemeister move if $WW^T = nI$ where $W_{i,j} = W_{1,1} (i,j = 1, \ldots, n)$. We call $W$ a type II matrix. When the entries of a type II matrix have absolute value one, it is a complex Hadamard matrix. Hadamard matrices and character tables of finite abelian groups are examples of complex Hadamard matrices.

From each type II matrix $W$, Nomura’s elegant construction gives the Bose-Mesner algebra of an association scheme [7], [4]. We call this algebra the Nomura algebra of $W$ and denote it $N_W$. In [4], Jaeger et al. showed that for any type II matrix $W$, the nonura algebras of $W$ and $W^T$ are formally dual. More importantly, they proved that if $W$ is a spin model then it belongs to $N_W$ and $N_W = N_W^T$ is formally self-dual. We are motivated to find type II matrices with non-trivial Nomura algebra, in the hopes of getting new dual pairs of association schemes and finding new spin models. In [A], Godsil and I found the type II matrices associated with various combinatorial objects. In [C], Hosoya and I determined the Nomura algebra of the type II matrices arising from conference graphs.

In [8], Suzuki posed a related problem: determine the type II matrices whose Nomura algebras contain a given association scheme. He proved that if the Hamming scheme $H(n, 3)$ is contained in the Nomura algebra of $W$ then $W$ is equivalent to the character table of the group $\mathbb{Z}_3^n$. In [I], Mune-masa and I generalized Suzuki’s result for Hamming schemes $H(n, q)$ ($n \geq 2$ and $q \geq 3$) and generalized Hamming schemes, that $W$ is equivalent to the tensor product of $n$ type II matrices. In [G], the spin models in generalized Hamming schemes were examined.

In [5], Jones constructed representation of braid groups and link invariants from spin models. In particular, the Jones’ polynomial arises from the
Spin models and four-weight spin models. Jones pairs yield link invariants under very simple conditions. We established the equivalence of four-weight spin models to the Jones pairs in which both matrices are type-II.

Furthermore, the tools we developed for Jones pairs in our paper gave new and often simpler proofs to existing results on the association schemes attached to spin models and four-weight spin models. From each $n \times n$ four-weight spin model, we assembled an $2n \times 2n$ type-II matrix and an $4n \times 4n$ symmetric spin model [B]. Finally, we showed that these type-II matrices of three different sizes supply a family of four association schemes which are related to each other in the form of subschemes and quotient schemes.

There are three notions of duality for association schemes: formal duality, Delsarte’s notion, and hyper-duality on the Terwilliger algebra of the association schemes. The Hamming schemes satisfy all three notions of duality, but in [I], we saw that they are not the Nomura algebra of type II matrices when $n \geq 2$ and $q \geq 3$. In the case of four-weight spin models, Godsil and I observed a connection of the duality of the Nomura algebras to braid groups.

Type II matrices and spin models also appear in quantum computing. Godsil and Roy observed that every pair of unbiased bases give a complex Hadamard matrix, and they constructed three mutually unbiased bases from every spin model [2]. Complex Hadamard matrices have applications in quantum information theory, harmonic analysis, operator theory and combinatorics. There are interests in the construction of families of complex Hadamard matrices, as well as the classification of complex Hadamard matrices of small sizes.

Forty years ago, Goethals and Seidel [3] showed that a strongly regular graph yields a Hadamard matrix if and only if it is of Latin-square type or negative Latin square type. In [H], we determined there are only three additional families of parameters when the adjacency algebra of a strongly regular graph contains a complex Hadamard matrix. These families are either regular two-graphs or neighbourhoods of regular two-graphs. In the same paper, we saw that only three distance regular covers of complete graphs give complex Hadamard matrices.

A construction by Diţă[1] gives most of the parametric families of complex Hadamard matrices in the catalogue of small complex Hadamard matrices compiled by Tadej and Życzkowski [9]. For the classification of small complex Hadamard matrices, it is useful to be able to check whether a new
construction gives complex Hadamard matrices of Diţă-type. In [F], we used Nomura algebra to get a necessary condition for a complex Hadamard matrix to be of Diţă-type. For instance, this condition allows us to show that a family of $12 \times 12$ complex Hadamard matrices constructed by Matolcsi et al. in [6] are not of Diţă-type.

Publication


REFERENCES

References


