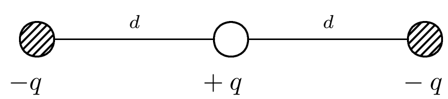
PHYS 2020: Homework 1 (due Monday Sept. 21)

Reading: Purcell & Morin, Chapters 1.1–1.6.

Problem 1 (10 points): The hydrogen atom is made from one proton and one electron, with an average separation distance of 5.29×10^{-11} m. Compute the Coulomb and gravitational forces between them. What is ratio between the Coulomb and gravitational forces? (This is why gravity is considered a very weak force.)

Problem 2 (10 points): Besides the proton and neutron, there are many other types of particles made from quarks that have been produced and studied in the laboratory. Consider a particle made from three quarks with charge $q = +\frac{2}{3}e$, each at the corner of an equilateral triangle with side length $d = 10^{-15}$ m. Compute the magnitude of the Coulomb force acting on each quark. (Note how massive the forces are. Recall that 10 N is about the weight of a 1 kg mass.) The fact that such a particle can exist, despite the massively repulsive Coulomb force between quarks, implies the existence of an even stronger attractive force between quarks that holds them together, known as the *strong force*.

Problem 3 (30 points): Consider the configuration consisting a $+q$ charge and two $-q$ charges aligned along the x-axis as follows:



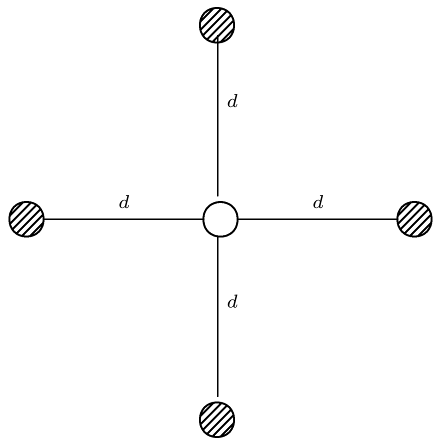
where the charges are separated by a distance d. In this position, the $+q$ charge is in equilibrium since the net force acting on it is zero.

(a) Suppose the $+q$ charge is moved perpendicularly by a vertical distance y. Now, what is the net force $\vec{F}$ acting on it?

(b) Next, suppose instead the $+q$ charge is moved parallel along the x-axis by a horizontal distance x. Now what is the net force $\vec{F}$ acting on it? Assume $|x| < d$.

Assuming the $-q$ charges are fixed, is the $+q$ charge in stable equilibrium at its initial position? Explain your reasoning.

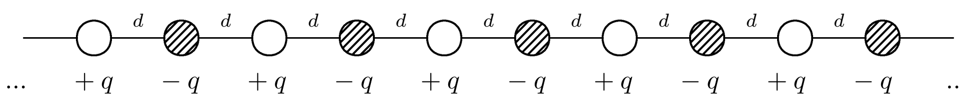
(c) Consider a new setup with four $-q$ charges surrounding a $+q$ as follows:



Is the $+q$ charge in a stable equilibrium here? Justify your reasoning.

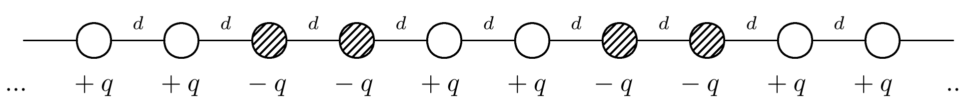
Problem 4 (20 points): A crystal is a periodic lattice of positively and negatively charged ions.

(a) Consider an infinite one-dimensional crystal of alternating charges $+q$ and $-q$, separated by distance d:



What is the potential energy per ion?

(b) Consider a similar setup with alternating *pairs of charges*, as follows:



What is the potential energy per ion?

Which crystal configuration is more stable (i.e. has the lowest potential energy per ion)?

Hint: You should encounter an infinite alternating series, which may be summed as follows:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ ... \ = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \ln(2) \qquad (1)$$

Bonus (5 points): Prove Eqn. (1) by Taylor expanding $\ln(1+x)$.