PHYS 2020: Homework 2 (due Monday Sept. 28)

Reading: Purcell & Morin, Chapters 1.7–1.8.

Problem 1 (20 points): Consider a straight line path L_1 from starting point $\vec{r}_1 = (0, 0, 0)$ to ending point $\vec{r}_2 = (1, 1, 1)$, shown in Fig. 1. Compute the work to move along L_1 for the following forces:

(a) $\vec{F} = -\begin{pmatrix} a\,x^2 \\ a\,y^2 \\ a\,z^2 \end{pmatrix}$, where a is a constant. **(5 points)**

(b) $\vec{F} = -\begin{pmatrix} b\,z \\ -b\,x \\ b\,y \end{pmatrix}$, where b is a constant. **(5 points)**

Now repeat your calculation along a new path L_2 from $\vec{r}_1$ to $\vec{r}_2$ along the curved line $y = z = x^2$. Comment on whether forces (a) and (b) are conservative or nonconservative. **(10 points)**

Problem 2 (10 points): Consider a cylinder of radius R and height L, aligned along the z axis, with charge density

$$\rho(s, z) = \begin{cases} a\,s^2 z & 0 < s < R \text{ and } 0 < z < L \\ 0 & \text{otherwise} \end{cases},$$

where a is a constant and s, z are the usual cylindrical coordinates. What is the total charge of the cylinder?

Problem 3 (30 points): Consider a circular wire ring (assumed to be infinitely thin) located in the x-y plane and centered at the origin. The ring has radius R and linear charge density λ. What is electric field $\vec{E}$ at a height h directly above the center of the ring, at the point $\vec{r} = (0, 0, h)$?

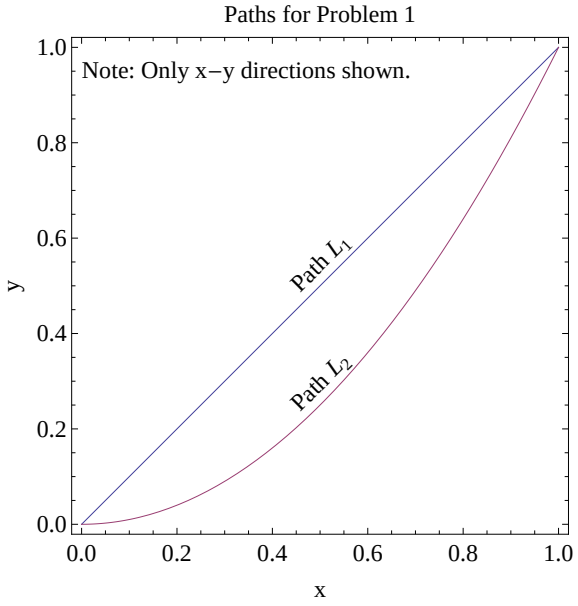

Figure 1: Paths L_1 and L_2 for Problem 1. Only the x and y directions are shown here. The z direction is not shown.