PHYS 2020: Homework 4 (due Monday Oct. 19)

Reading: Purcell & Morin, Chapters 2.1–2.6.

Problem 1 (20 points): Consider a thin circular ring of radius R and uniform linear charge density λ. What is the potential at the center of the ring? *Hint:* You already computed the electric field from the ring in HW 2, problem 3.

Problem 2 (10 points): Consider two point charges, q and $-q$, located at $\vec{r}_1 = (0, 0, 0)$ and $\vec{r}_2 = (d, 0, 0)$, respectively. What is the potential difference $\phi_{43} = \phi(\vec{r}_4) - \phi(\vec{r}_3)$, where $\vec{r}_3 = (0, d, 0)$ and $\vec{r}_4 = (d, d, 0)$?

Problem 3 (20 points): Consider an electric field

$$\vec{E}(\vec{r}) = a \begin{pmatrix} 6xy \\ 3x^2 - 3y^2 \\ 0 \end{pmatrix}, \tag{1}$$

where a is a constant. What is the potential

$$\phi(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{ds} \cdot \vec{E}(\vec{r}\,') \tag{2}$$

relative to the reference position $\vec{r}_0 = (0, 0, 0)$? Using your result for ϕ, verify that $-\vec{\nabla}\phi$ gives Eq. (1).

Hint #1: Don't forget to use *primed* variables as your integration variables. (Remember $\vec{r} = (x, y, z)$ is the end point of the integral and $\vec{r}\,'$ is the position along the line integral.) So, make sure you set $\vec{ds} = (dx', dy', dz')$ and plug in $\vec{E}(x', y', z')$ into Eq. (2).

Hint #2: Evaluate Eq. (2) first along the x-direction from 0 to x, then along the y-direction from 0 to y, and lastly along the z-direction from 0 to z.

Problem 4 (20 points): Purcell & Morin, exercise 2.41.

Problem 5 (10 points): Compute the gradient $\vec{\nabla} f$ of the following functions:

(a) $f(\vec{r}) = 2(xy + y^2 - xz)$. Express your result in Cartesian coordinates. **(5 points)**

(b) $f(\vec{r}) = 1/r^4$. Express your result in spherical coordinates. **(5 points)**