PHYS 2020: Homework 5 (due Monday Oct. 26)

Reading: Purcell & Morin, Chapters 2.7. Optional reading, 2.8–2.18.

Problem 1 (20 points): Consider a solid sphere of radius R and uniform charge density ρ. Let r be the distance from the center of the sphere.

(a) What is the electric field $\vec{E}(r)$? **(5 points)**

(b) What is the potential $\phi(r)$? **(5 points)**

(c) What is the potential difference $\Delta\phi$ between the center and the outer surface of the sphere? **(5 points)**

(d) Using your expression for $\phi(r)$ in part (b), verify that $\vec{E} = -\vec{\nabla}\phi$. **(5 points)**

Problem 2 (20 points): Consider an infinite solid cylinder of radius R and uniform charge density ρ. Let s be the distance from the center of the sphere.

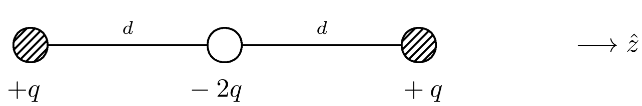
(a) What is the electric field $\vec{E}(s)$? **(5 points)**

(b) What is the potential $\phi(s)$? Define the zero of your potential relative to the surface of the cylinder ($s = R$) **(5 points)**

(c) What is the potential difference $\Delta\phi$ between the center and the outer surface of the cylinder? **(5 points)**

(d) Using your expression for $\phi(s)$ in part (b), verify that $\vec{E} = -\vec{\nabla}\phi$. **(5 points)**

Problem 3 (20 points): Consider a charge configuration aligned along the z-axis, shown as follows:



Evaluate the potential $\phi(\vec{r})$ in the limit $d \ll r$, and express your results in terms of spherical coordinates (r, θ). Show that the leading term is given by the *quadrupole* moment.

Problem 4 (10 points): Consider an open hemispherical bowl with radius R and surface charge density σ. The lip of the bowl forms a circle C of radius R. Show that the potential at any point level with and within C has a constant potential, given by $\phi = \frac{\sigma R}{2\epsilon_0}$.

Hint: No calculus is needed. First, consider a spherical shell, and then use the superposition principle.