PHYS 2020: Homework 6 (due Monday Nov. 2)

Reading: Purcell & Morin, Chapters 3.1–3.4.

Problem 1 (15 points): Consider a solid sphere of uniform charge density ρ and radius R. Let $\vec{r} = (x, y, x)$ be the position vector relative to the center of the sphere.

(a) In class, we derived the electric field $\vec{E}$ for the sphere in spherical coordinates. Write down this result for $\vec{E}$ (no need to rederive it) and express your result in *Cartesian coordinates*. **(5 points)**

(b) Show that $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$ for $r < R$, while $\vec{\nabla} \cdot \vec{E} = 0$ for $r > R$. **(5 points)**

(c) Show that $\vec{\nabla} \times \vec{E} = 0$ for any r.

Problem 2 (15 points): Purcell & Morin, problem 3.33.

Problem 3 (10 points): Purcell & Morin, problem 3.38.

Problem 4 (20 points): Consider an infinite conducting plane in the x-y plane. An infinite wire with uniform linear charge density λ is located parallel to the y axis at a height $z = d$ above the conducting plane.

(a) Sketch the location and charge distribution of the mirror charge. **(5 points)**

(b) Sketch the resulting electric field $\vec{E}$ for region above the plane, i.e. $z > 0$. **(5 points)**

(c) What is the electric field $\vec{E}$ at the surface of the plane ($z = 0$)? **(5 points)**

(d) What is the surface charge density σ on the surface of the plane? **(5 points)**