Review of chapter 1 : Important concepts

(1) Coulomb's law ; including vector form.

Forces between point charges

(2) Work, line integrals: $W = -\int \vec{ds} \cdot \vec{F}$

(i) General case : arbitrary force $\vec{F}(x,y,z)$ along arbitrary line.

$$\vec{ds} = (dx, dy, dz) = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

- Use equation for line to express y, z in terms of x. $y = y(x)$, $z = z(x)$.

$$\vec{F} = \vec{F}(x,y,z) \rightarrow \underbrace{\vec{F}(x, y(x), z(x))}_{\text{function of } x \text{ only}}$$

- Compute dy, dz in terms of dx.

$$dy = y'(x)\,dx, \quad dz = \frac{dz}{dx}\,dx = z'(x)\,dx$$

- Plug in to get W: express everthing in terms of one variable (say x)

$$W = -\int \left(dx\,F_x(x,y,z) + dy\,F_y(x,y,z) + dz\,F_z(x,y,z) \right)$$

$$= -\int dx \Big[F_x(x, y(x), z(x)) + F_y(x, y(x), z(x))\,y'(x)$$

$$+ F_z(x, y(x), z(x))\,z'(x) \Big]$$

(ii) Special cases with symmetry.

e.g. Spherical coords, for line along radial direction $\vec{ds} = dr\,\hat{r}$

Cylindrical coords, for line along s direction
$\vec{ds} = ds\,\hat{s}$

— " — for line along z direction,
$\vec{ds} = dz\,\hat{z}$

(3) Electrostatic forces are conservative, any path between $\vec{r}_1$ and $\vec{r}_2$ gives same result.

(4) Potential energy for point charge configuration

$$U = \frac{1}{2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$$

(5) Continuous charge distributions

(i) Line charge (e.g. charged wire)

λ = linear charge density

(ii) Surface charge (e.g. plane)

σ = surface charge density

(iii) Volume charge (e.g. solid sphere)

ρ = charge density

(6) Total charge for continuous charge distributions

Divide charge distribution into tiny regions dq

$Q = \int dq$.

(i) volume charge $\quad dq = \rho \, dV$

Cartesian coords $\quad dV = dx\,dy\,dz$

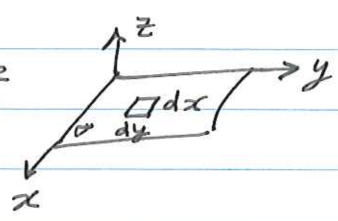
Spherical coords $\quad dV = r^2 \sin\theta \, dr\, d\theta\, d\phi$

Cylindrical coords $\quad dV = s\, ds\, d\theta\, dz$

(ii) surface charge $\quad dq = \sigma \, dA$

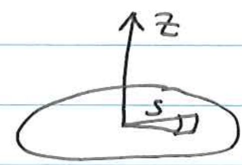
Cartesian coords $\quad dA = dx\,dy$

e.g. flat plane



Cylindrical coords $\quad dA = s\, ds\, d\theta$

e.g. flat disk



Spherical coords

e.g. spherical shell

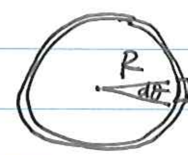
$dA = R^2 \sin\theta \, d\theta \, d\phi$

(iii) line charge $\quad dq = \lambda \, d(\text{length})$

Cartesian coords
e.g. straight wire

$dq = \lambda \, dx$

Polar coords
e.g. ring



$dq = \lambda R \, d\theta$

(7) Electric field $\vec{E}$

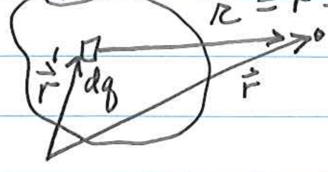
(i) Point charge q : $\quad \vec{E}(\vec{r}) = \dfrac{q}{4\pi\varepsilon_0 r^2}\hat{r}$

(ii) N point charges

(iii) Continuous charge distribution

$$\vec{E}(\vec{r}) = \int \frac{1}{4\pi\varepsilon_0} \frac{dq}{\mathscr{r}^2} \hat{\mathscr{r}}$$

where $\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$ and dq given above.
$\vec{r}$ = where you are "measuring" $\vec{E}$
$\vec{r}\,'$ = where dq is located.



$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$ = distance from dq to where you measure $\vec{E}$.

(8) Electric flux & surface integrals

$$\Phi_E = \int d\vec{A} \cdot \vec{E}$$

$d\vec{A}$ has magnitude $|d\vec{A}| = dA$ = same as for integrating over surface charges.

$d\vec{A}$ has direction pointing normal to surface.

(9) Gauss's Law:

$$\Phi_E = \int d\vec{A} \cdot \vec{E} = \frac{Q_{encl}}{\varepsilon_0}$$

Applications: Find Gaussian surface where $\vec{E}$ is normal and $|\vec{E}|$ is constant over surface.

$$\Rightarrow \int d\vec{A} \cdot \vec{E} = A\,E = \frac{Q_{encl}}{\varepsilon_0} \Rightarrow E = \frac{Q_{encl}}{A\,\varepsilon_0}$$