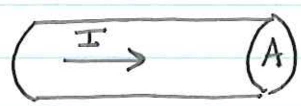
IV. Electric currents

So far we have considered electro<u>statics</u> — charges are stationary.

Now we consider moving charges (in a conductor).

Current I = charge-per-unit-time flowing through a surface area A.

e.g. conducting wire. A = cross sectional area of wire.

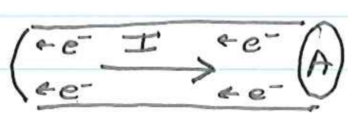


We are (mainly) going to consider <u>steady state</u> currents.

→ I is constant in time.

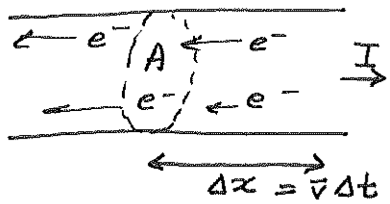
I has units $\dfrac{\text{Coulomb}}{\text{second}} = 1$ ampere.

By convention, electron has negative charge.
I is opposite to the way electrons flow.



I is the flux of electrons moving through A (in the opposite direction)

Suppose electrons have average velocity $\bar{v}$. In time Δt, all e^- within a distance $\Delta x = \bar{v}\,\Delta t$ will flow through the ~~[illegible]~~ area A.



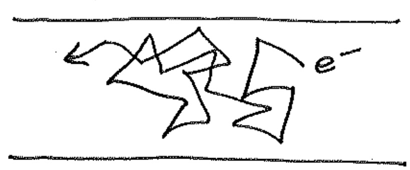
Let n_e be the number density of electrons
number of electrons/volume.

Then the charge of the electrons flowing through A is:
(in time Δt)

$$\Delta Q = e\, n_e\, \text{Volume} = e\, n_e\, \Delta x\, A = e\, n_e\, \bar{v}\, \Delta t\, A$$

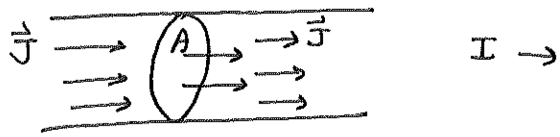
The current is $\dfrac{\Delta Q}{\Delta t} = I = e\, n_e\, A\, \bar{v}$

The individual velocities of electrons is typically much larger than $\bar{v}$.



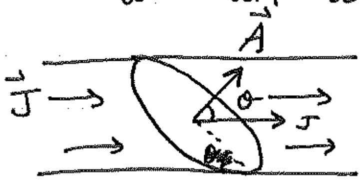
<u>Current density</u>

$\vec{J}$ is the current density. Current-per-unit-area. It is a vector pointing in the direction that charge is flowing.



For an area A normal to $\vec{J}$, $I = JA$.

But we can also consider areas not normal to $\vec{J}$.



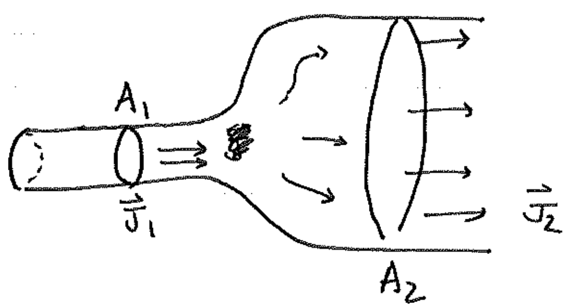
Define area vector $\vec{A}$.

$$I = \vec{J} \cdot \vec{A} = J A_\perp$$

where $A_\perp = A\cos\theta$ = cross sectional area perpendicular to $\vec{J}$.

Note: I doesn't depend on how we draw the surface area A.

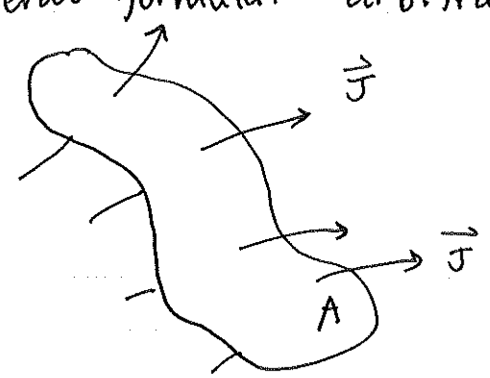
Consider a wire of variable width:



Note the current I must be same through A_1 and A_2 (otherwise charge would pile up)

but $J_1 = \frac{I}{A_1} > J_2 = \frac{I}{A_2}$.

More general formula: arbitrary $\vec{J}$ and arbitrary surface



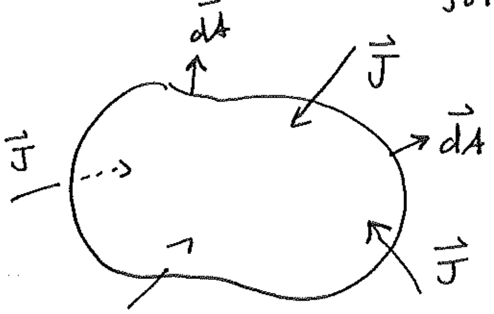
$$I = \int_A d\vec{A} \cdot \vec{J}$$

Same flux formula as Gauss's law.

What about a closed surface?

Gauss's law: $\Phi_E = \int_A d\vec{A} \cdot \vec{E} = \frac{Q_{encl}}{\varepsilon_0}$

enclosed charge acts as a source / sink for $\vec{E}$ "flow"



closed surface

this is the case where $\int_A d\vec{A} \cdot \vec{J} < 0$

If we have a net flux for $\vec{J}$ ($\int d\vec{A} \cdot \vec{J} \neq 0$), then charge has a net flow into a closed region.

Charges will accumulate as a function of time $\rightarrow$ not a steady state.

Steady state: $\int_A d\vec{A} \cdot \vec{J} = 0$ current in = current out

for a closed surface A.

For not a steady state $\int_A d\vec{A} \cdot \vec{J} = -\frac{dQ_{encl}}{dt}$

$$\int_A d\vec{A} \cdot \vec{J} = -\frac{dQ_{encl}}{dt}$$

This is called the continuity equation. It is a consequence of conservation of charge.

If charge is flowing into a closed surface, it must pile up inside.

Recall: Gauss's law had an integral & differential form:

$$\int d\vec{A} \cdot \vec{E} = \frac{Q_{encl}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int dV \rho \quad \rightarrow \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

The continuity equation also has a differential form:

$$\int d\vec{A} \cdot \vec{J} = -\frac{d}{dt} Q_{encl} = -\frac{d}{dt} \int dV \rho = -\int dV \frac{d\rho}{dt}$$

$$\rightarrow \quad \vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt}$$

if the current density has a divergence at a point $\vec{r}$, then the charge density must be decreasing at that point.

<u>Electrical conductivity</u>

A current is moving charges. Charges in a conductor move because they feel an electric field.

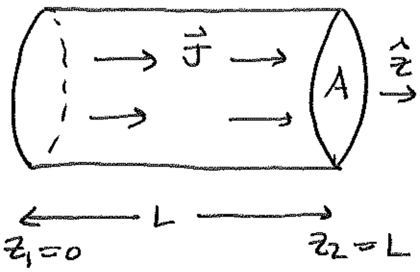
$$\vec{J} = \sigma \vec{E}$$ This is a form of Ohm's law.

It is an empirical relation that $\vec{J}$ is proportional to $\vec{E}$. σ is the <u>conductivity</u>. Larger σ, more current flows for a given $\vec{E}$.

The inverse, $\rho = 1/\sigma$, is the resistivity.

For electrostatics, $\vec{E} = 0$ in a conductor. (no current)
If there is a current, then $\vec{E}$ is nonzero.

Length of wire of cross section A:



If A is constant along the wire, then $\vec{J} = I/A\,\hat{z}$ everywhere inside wire.

What is the potential difference across the wire?

$$\Delta\phi = -\int_{z_2}^{z_1} \vec{ds}\cdot\vec{E} = -\int_{L}^{0} dz\, \underbrace{\rho J}_{E=\rho J=\frac{1}{\sigma}J} = J\rho L$$

$$= I\left(\frac{\rho L}{A}\right)$$

$\Delta\phi$ is also known as the <u>voltage</u> V.

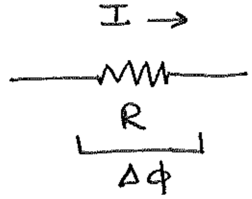
So we have $V = IR$, where $R = \dfrac{\rho L}{A}$ is the <u>resistance</u>. R has units of ohms.

Note: ρ (and σ) depends only on the properties of the material.

e.g. copper (at room temp) has $\rho = 1.58\times10^{-8}$ ohm m

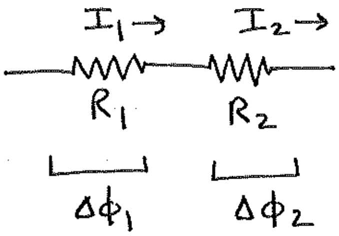
R depends on the geometry of the object.

Resistors:



potential difference across R is $\Delta\phi$ $(=V)$

Resistors in ~~parallel~~ series:



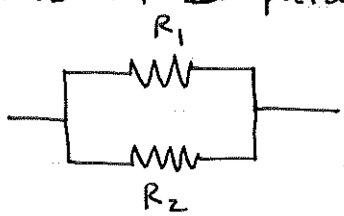
$$\Delta\phi_1 = V_1 = I_1 R_1$$
$$\Delta\phi_2 = V_2 = I_2 R_2$$

total potential difference:

$$V = \Delta\phi = V_1 + V_2 = I_1 R_1 + I_2 R_2 = I R$$

since $I = I_1 = I_2 \rightarrow$ $\underline{R = R_1 + R_2}$

Resistors in parallel:



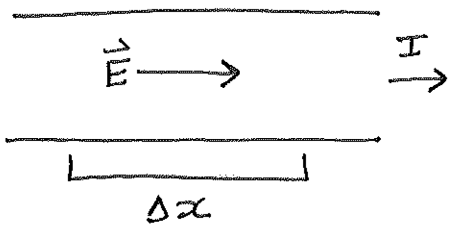
now $V = V_1 = V_2$
$I = I_1 + I_2$

So $$I = \frac{V}{R} = I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\Rightarrow \underline{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Energy dissipation by current flow

Current in a conductor:



A given charge ΔQ experiences a force $F = \Delta Q E$.
The work on ΔQ as it travels a distance Δx is:

$$W = -\int \vec{F} \cdot d\vec{s} = -F\,\Delta x = -\Delta Q\, E\, \Delta x$$

Energy is lost (dissipated) as current flows.

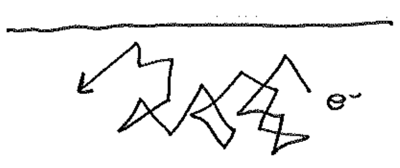
Power (energy/time) for energy loss is

$$P = -\frac{W}{\Delta t} = \frac{\Delta Q}{\Delta t} E\, \Delta x = I\, E\, \Delta x$$

Ohm's law: $E = \rho J = \rho \dfrac{I}{A}$

$$P = I^2 \left(\frac{\rho\, \Delta x}{A} \right) = I^2 R = IV = \frac{V^2}{R}$$

Energy is dissipated as heat



collisions of e^- with ions in conductor heats up the conductor.