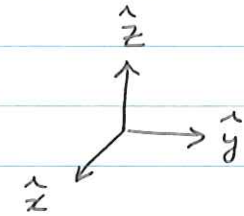
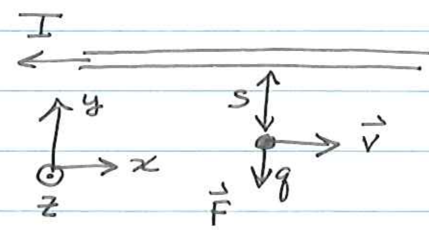
VI. Magnetic forces

First, recall cross product of unit vectors:



$$\hat{x} \times \hat{y} = \hat{z} = -\hat{y} \times \hat{x}$$
$$\hat{y} \times \hat{z} = \hat{x} = -\hat{z} \times \hat{y}$$
$$\hat{z} \times \hat{x} = \hat{y} = -\hat{x} \times \hat{z}$$

We showed that the magnetic force from a wire on a moving charge q is:



$$\vec{F} = -\frac{qvI}{2\pi\varepsilon_0 c^2 s}\hat{y}$$

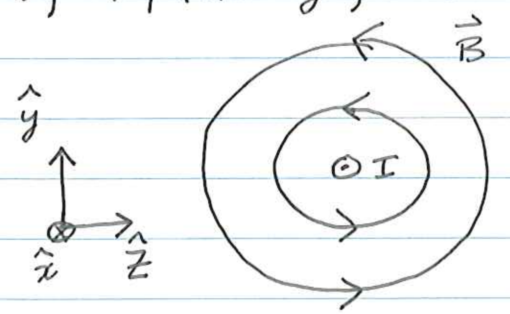
Lorentz force is: $\vec{F} = q\vec{v} \times \vec{B}$

Here $\vec{v} = v\hat{x}$, so we need $\vec{B} = \dfrac{I}{2\pi\varepsilon_0 c^2 s}\hat{z}$

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{qvI}{2\pi\varepsilon_0 c^2 s}\underbrace{\hat{x} \times \hat{z}}_{=-\hat{y}}$$

$$B = \frac{\mu_0 I}{2\pi s}, \quad \mu_0 = \frac{1}{\varepsilon_0 c^2} = \text{permeability of free space.}$$

By symmetry, $\vec{B}$ points in a circle around the wire



current flowing out of page.

$\vec{B}$ lines have no beginning/end (no magnetic charges)

We're going to learn 3 ways to compute $\vec{B}$.
Similar to computing $\vec{E}$ in three ways:

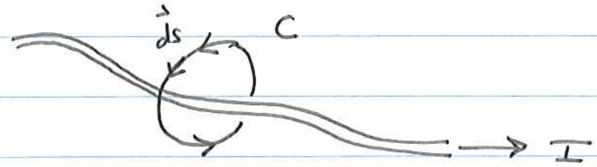
(1) Gauss's law (only for special cases with symmetry)

(2) General formula:
$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 \mathscr{r}^2}\,\hat{\mathscr{r}}$$

(3) From the potential: first find ϕ, then take $\vec{E} = -\vec{\nabla}\phi$.

Ampere's law: (analog of Gauss's law for currents)

Current I carried by a wire; causes a $\vec{B}$ field.

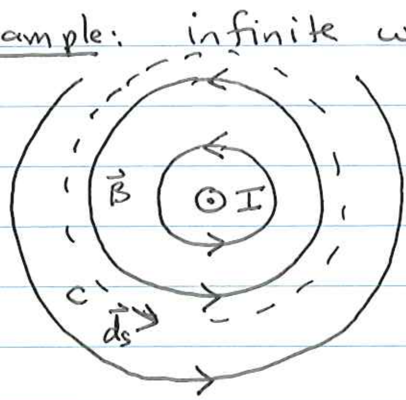


Draw a closed loop C around the wire (Amperian loop)

$$\boxed{\text{Ampere's law: } \int_C \vec{ds}\cdot\vec{B} = \mu_0 I_{encl}}$$

I_{encl} is the current flowing through the loop.
Here $I_{encl} = I$.

example: infinite wire, current flowing out of the page.



Draw Amperian loop as circle of radius s.

$\vec{ds}$ points along the loop in same direction as $\vec{B}$. $\vec{ds}\cdot\vec{B} = B\,ds$

Also $B = |\vec{B}|$ is constant along loop by symmetry.

$$\int_C \vec{ds} \cdot \vec{B} = \int_C ds\, B = B \int_C ds = B \cdot 2\pi S$$

↑ $\vec{ds}, \vec{B}$ point in same direction

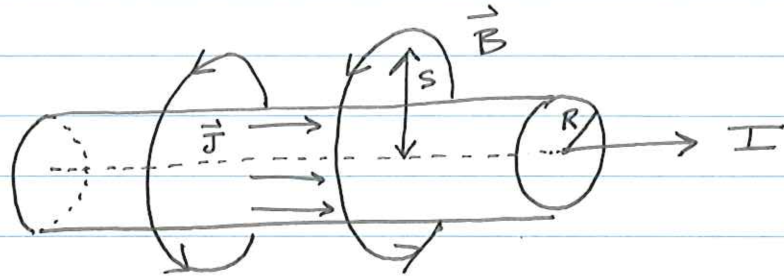
↑ $B = \text{const}$ around C

↑ length of C = circumference $2\pi S$

So we have $\int_C \vec{ds} \cdot \vec{B} = B\, 2\pi S = \mu_0 I_{enc} = \mu_0 I$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi S}}$$

example: thick wire of radius R, carrying current I. What is $\vec{B}(s)$, where s = distance from center of the wire?



Case 1: $S > R$, same as above:

$$B = \frac{\mu_0 I_{enc}}{2\pi S} = \frac{\mu_0 I}{2\pi S}$$

Case 2: $S < R$.

cross section of wire.

Note: $\vec{J}$ is constant ~~through~~ through the wire. $J = I/\pi R^2 = \frac{I}{A}$

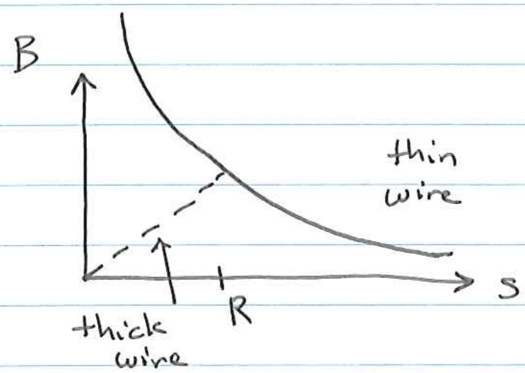
Only a part of the total current is enclosed inside C.

$$I_{enc} = J \cdot \text{Area of } C$$
$$= J \cdot \pi S^2$$

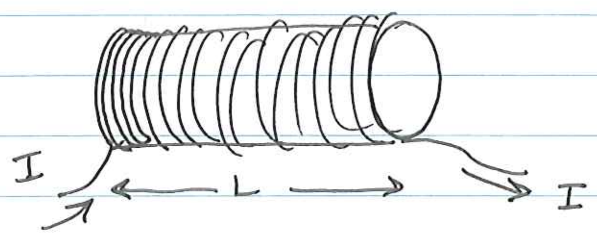
So $I_{enc} = J \pi s^2 = \frac{I}{\pi R^2} \pi s^2 = \frac{I s^2}{R^2}$

$$B = \frac{\mu_0 I_{enc}}{2\pi s} = \frac{\mu_0 I s^2}{2\pi s R^2} = \frac{\mu_0 I s}{2\pi R^2}$$

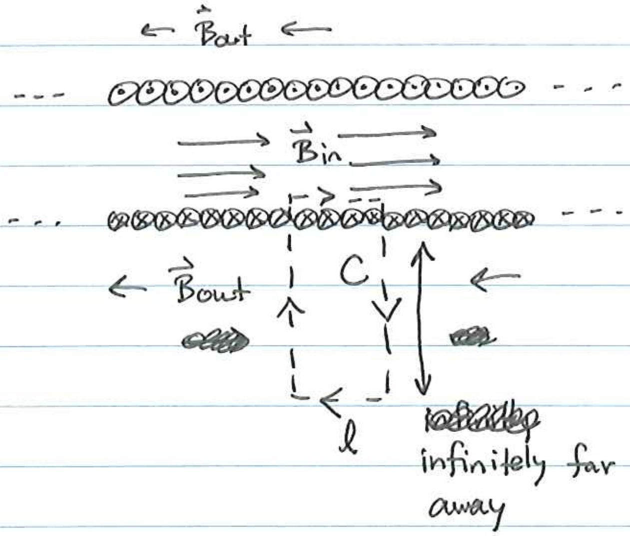
Final result: $B = \begin{cases} \dfrac{\mu_0 I}{2\pi s} & s > R \\[2ex] \dfrac{\mu_0 I s}{2\pi R^2} \end{cases}$



example: solenoid



Solenoid with length L and N loops carrying current I.



For an ideal solenoid ($L \to \infty$), $\vec{B}$ must point parallel to the solenoid axis (can't point radially, since $\vec{B}$ lines can't end)

Ampere's law:

$$\int_C \vec{ds} \cdot \vec{B} = B_{in}\, l - B_{out}\, l = \mu_0 I_{encl}$$

Bottom part of loop is infinitely far away: $B_{out} \to 0$.

$$B_{in}\, l = \mu_0 I_{encl}$$

Suppose $I_{encl} = n I$, $n=$ number of windings enclosed in loop.

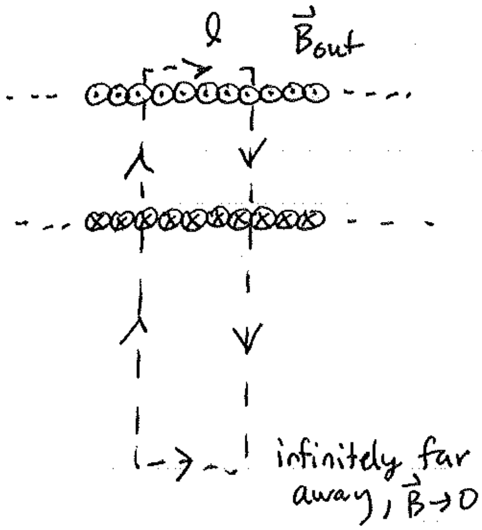
$$B_{in} = \frac{\mu_0 n I}{l}$$

n/l is the number of windings per unit length.

So $n/l = \dfrac{N}{L} = \dfrac{\text{total number of windings}}{\text{total length}}$.

Magnetic field inside: $\boxed{B = \dfrac{\mu_0 N I}{L}}$ constant

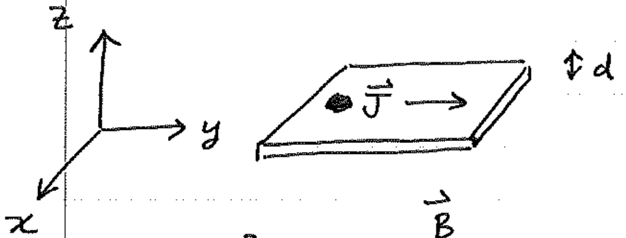
Consider a different Amperian loop to find Bout just outside the solenoid:



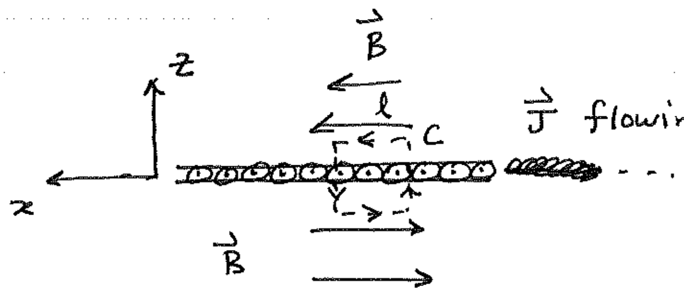
$$B_{out}\, l = \mu_0 I_{encl} = 0$$

So we have: $B = \begin{cases} \frac{\mu_0 N I}{L} & \text{inside} \\ 0 & \text{outside} \end{cases}$

<u>example</u>: current sheet, infinite sheet of thickness d with constant current density $\vec{J}$.



$\vec{B}$ must point parallel to sheet



$\vec{J}$ flowing out of page.

Amperian loop: $\int_C \vec{ds} \cdot \vec{B} = 2 \ell B = \mu_0 I_{encl}$

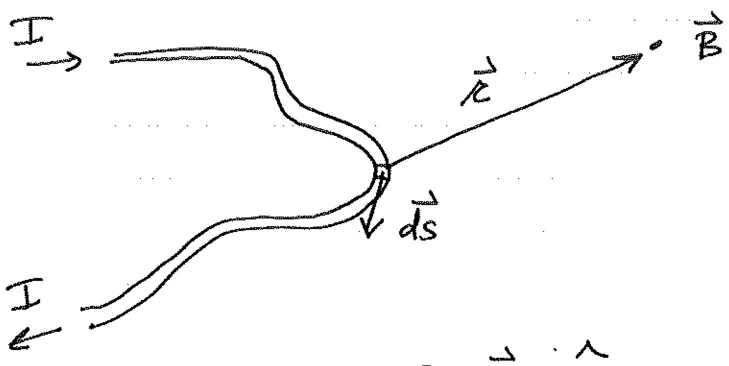
$$= \mu_0 J \ell d$$

$$\Rightarrow \quad B = \frac{\mu_0 J d}{2}$$

putting in the direction: $\vec{B} = \begin{cases} \frac{\mu_0 J d}{2} \hat{x} & z > 0 \text{ (above)} \\ -\frac{\mu_0 J d}{2} \hat{x} & z < 0 \text{ (below)} \end{cases}$

Biot-Savart Law

General formula for computing $\vec{B}$ from a current-carrying wire



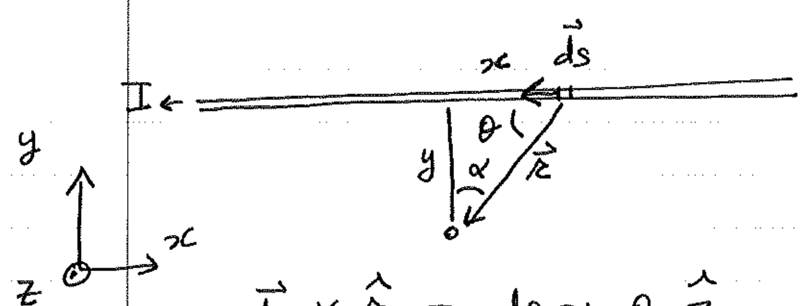
$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{ds} \times \hat{r}}{r^2}$$

$\vec{r}$ points from segment to where $\vec{B}$ is measured.

Integrate over length of wire.

$\vec{ds}$ points in same direction as I.

example: infinite straight wire



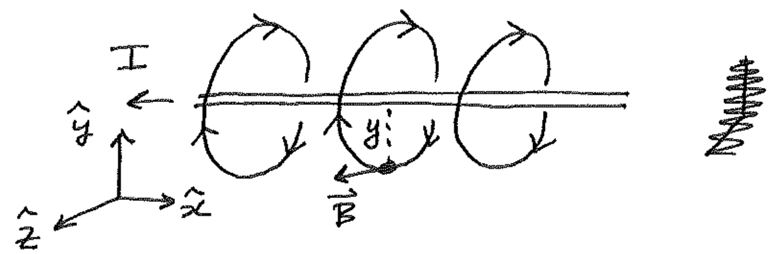
$$\vec{ds} \times \hat{r} = ds \sin\theta \, \hat{z}, \quad ds = dx, \quad \sin\theta = \frac{y}{r}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{dx \sin\theta \, \hat{z}}{(x^2+y^2)} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} dx \frac{y}{(x^2+y^2)^{3/2}} \hat{z}$$

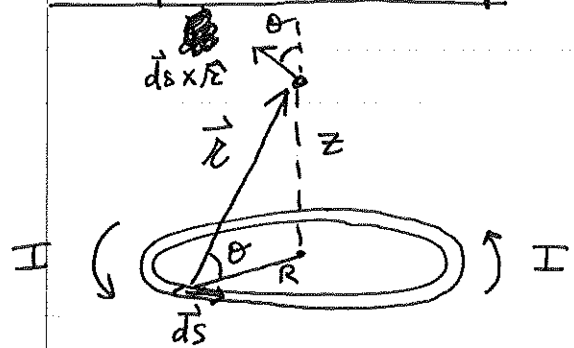
Trig sub: $x = y \tan\alpha$, $dx = y \sec^2\alpha \, d\alpha$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \hat{z} \underbrace{\int_{-\pi/2}^{\pi/2} d\alpha \frac{y^2 \sec^2\alpha}{y^3 \sec^3\alpha}}_{= \frac{1}{y} \int_{-\pi/2}^{\pi/2} d\alpha \cos\alpha = \frac{2}{y}}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi y}\,\hat{z}$$



example: current loop



Only $\hat{z}$ component of $\vec{B}$ will be non zero.

$$(\vec{ds} \times \hat{r})_z = ds\cos\theta$$

$$r = \sqrt{R^2+z^2}$$

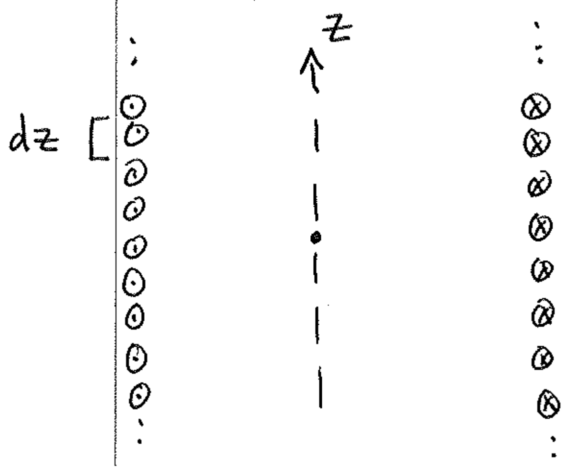
$$B_z = \int \frac{\mu_0 I}{4\pi}\,\frac{ds\cos\theta}{(R^2+z^2)^2}$$

Now $\cos\theta = \dfrac{R}{\sqrt{R^2+z^2}}$, $ds = R\,d\phi$

$$B_z = \frac{\mu_0 I}{4\pi}\int \frac{R\,d\phi\;R}{(R^2+z^2)^{3/2}} = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}\,\hat{z}$$

example: solenoid



Tower of N loops with length L

Consider a segment of length dz. Current in this segment is

$$dI = \text{current} = \frac{\text{current}}{\text{length}} \times \text{length}$$

$$= \frac{N\,I}{L} \cdot dz$$

$\vec{B}$ field from each segment dz is:

$$d\vec{B} = \frac{\mu_0\, dI\, R^2}{2\left(R^2 + z^2\right)^{3/2}}\,\hat{z}$$

Total $\vec{B}$ is:

$$\vec{B} = \int_{-\infty}^{\infty} \frac{\mu_0\, dI\, R^2}{2\left(R^2+z^2\right)^{3/2}}\,\hat{z} = \frac{\mu_0 N I}{2L} \int_{-\infty}^{\infty} dz\, \frac{R^2}{\left(R^2+z^2\right)^{3/2}}$$

↑ ideal solenoid

$N \to \infty$, $L \to \infty$, but

N/L is fixed

(loops per unit length)

Trig sub: $z = R\tan\theta$

$dz = R\sec^2\theta\, d\theta$

$$\vec{B} = \frac{\mu_0 N I}{2L} \int_{-\pi/2}^{\pi/2} d\theta\, \cos\theta\,\hat{z} = \frac{\mu_0 N I}{L}\,\hat{z}$$

Same result as before using Ampere's law

Maxwell's equations for magnetostatics (steady state currents)

Recall for electrostatics we had:

$$\int_S d\vec{A} \cdot \vec{E} = \frac{Q_{encl}}{\varepsilon_0}$$ for a closed surface S (Gauss's law)

$$\int_C d\vec{s} \cdot \vec{E} = 0$$ for a closed loop C (conservative force)

These corresponded to the equations in differential form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \times \vec{E} = 0$$

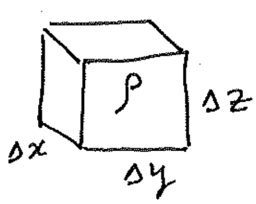
The corresponding equations for magnetostatics are

$$\int_S d\vec{A} \cdot \vec{B} = 0$$ no magnetic charges

$$\int_C d\vec{s} \cdot \vec{B} = \mu_0 I_{encl}$$ Ampere's law.

Let's derive the equations in differential form:
Recall what we did before for $\vec{E}$

Gauss's law for a tiny box $\qquad Q_{encl} = \rho\, \Delta x\, \Delta y\, \Delta z = \rho\, \Delta V$



$$\int d\vec{A} \cdot \vec{E} = \vec{\nabla} \cdot \vec{E}\, \Delta x\, \Delta y\, \Delta z = \frac{\rho\, \Delta x\, \Delta y\, \Delta z}{\varepsilon_0}$$

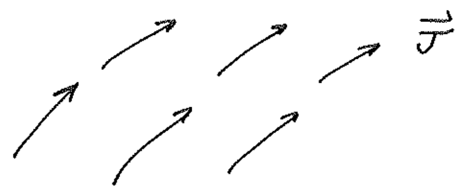
$$\Rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Apply same logic to $\vec{B}$. ($Q_{encl} \to 0$ since no magnetic charges)

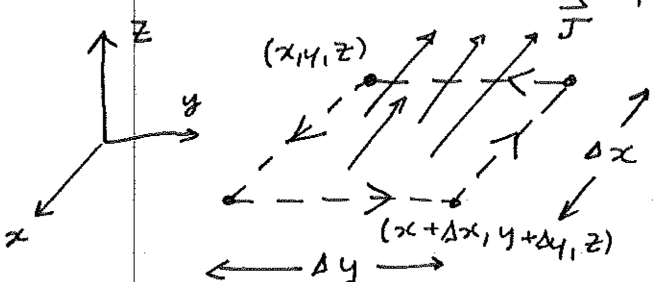
$$\int d\vec{A} \cdot \vec{B} = \vec{\nabla} \cdot \vec{B}\, \Delta x \Delta y \Delta z = 0 \Rightarrow \underline{\vec{\nabla} \cdot \vec{B} = 0}.$$

Next, use Ampere's law:

Consider $\vec{B}$ generated by a general current density $\vec{J}(\vec{r})$



Draw tiny Amperian loop C in x-y directions, i.e. normal to $\hat{z}$



$$I_{encl} = J_z A = J_z \Delta x \Delta y$$

only J_z component flows through the loop

$$\int_C d\vec{s} \cdot \vec{B} = B_x(x,y,z)\Delta x + B_y(x+\Delta x, y, z)\Delta y$$

$$- B_x(x, y+\Delta y, z)\Delta x - B_y(x,y,z)\Delta y$$

$$= -\frac{\partial B_x}{\partial y}\Delta x \Delta y + \frac{\partial B_y}{\partial x}\Delta x \Delta y$$

$$= \mu_0 I_{encl} = \mu_0 J_z \Delta x \Delta y$$

So $$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \mu_0 J_z$$

Free to draw any Amperian loop we want.
Also consider loops in the x-z & y-z directions.

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu_0 J_x$$

$$\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} = \mu_0 J_y$$

Put all three relations together:

$$\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} ,\ \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} ,\ \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right)$$

$$= \mu_0 \left(J_x, J_y, J_z \right)$$

$$\Rightarrow \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

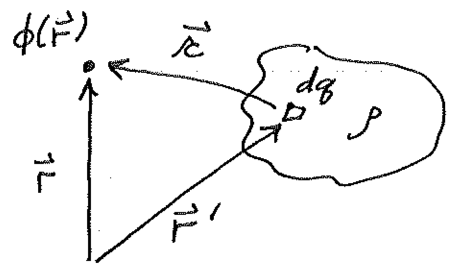
Maxwell's equations for Magnetostatics:

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

Vector potential

Recall: electric potential ϕ (also called scalar potential since ϕ is just a number, no direction)

Useful way to compute $\vec{E}$: first compute $\phi(\vec{r})$



$$\phi(\vec{r}) = \int \frac{dq}{4\pi\varepsilon_0 \mathscr{r}} = \int dV \frac{\rho}{4\pi\varepsilon_0 |\vec{r}-\vec{r}\,'|}$$

$$dq = \rho \, dV$$
$$\mathscr{r} = |\vec{r} - \vec{r}\,'|$$

Then $\vec{E} = -\vec{\nabla}\phi$. We only care about gradient (slope) of ϕ, free to shift ϕ by overall constant.

Recall also Maxwell's equations for $\vec{E}$: $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

$\vec{\nabla}\times\vec{E} = 0$ automatically satisfied

$$\Rightarrow \quad \nabla^2\phi = -\frac{\rho}{\varepsilon_0}.$$ Solution to this is given above.

$$= \left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}\right)$$

We can also define a potential for $\vec{B}$.

$\vec{A}$ is called the <u>vector potential</u>. It is defined by

$$\boxed{\vec{\nabla}\times\vec{A} = \vec{B}}$$

i.e. we can compute $\vec{A}$, then take the curl to get $\vec{B}$.

We only care about the curl of $\vec{A}$. Free to set the divergence to anything $\rightarrow \vec{\nabla}\cdot\vec{A} = 0$.

$$= \left(\frac{\partial}{\partial x}(\vec{\nabla}\cdot\vec{A}) - \nabla^2 A_x, \frac{\partial}{\partial y}(\vec{\nabla}\cdot\vec{A}) - \nabla^2 A_y, \frac{\partial}{\partial z}(\vec{\nabla}\cdot\vec{A}) - \nabla^2 A_z \right)$$

$$= \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$

So we have: $\vec{\nabla}\times\vec{B} = \vec{\nabla}\times\vec{\nabla}\times\vec{A} = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$

Use $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ and $\vec{\nabla}\cdot\vec{A} = 0$

$$\nabla^2\vec{A} = (\nabla^2 A_x, \nabla^2 A_y, \nabla^2 A_z) = -\mu_0\vec{J}$$

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} = -\mu_0 J_x$$

$$\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} = -\mu_0 J_y$$

$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} = -\mu_0 J_z$$

Same solution for A_i as for ϕ, but with

$$-\rho/\varepsilon_0 \longrightarrow -\mu_0 J_i \qquad (i = x, y, z)$$

$$A_i = \int dV \frac{J_i \mu_0}{4\pi r}$$

$$\boxed{\vec{A}(\vec{r}) = \int dV \frac{\mu_0 \vec{J}}{4\pi r}}$$

$$\vec{A}(\vec{r}) = \iiint dx'dy'dz' \frac{\mu_0 \vec{J}(\vec{r}\,')}{4\pi \,\mathscr{r}}$$

where $\mathscr{r} = |\vec{\mathscr{r}}| = |\vec{r} - \vec{r}\,'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$

Now compute $\vec{B} = \vec{\nabla} \times \vec{A}$. Since $\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ only acts on $\vec{r} = (x, y, z)$, not $\vec{r}\,' = (x', y', z')$, $\vec{\nabla}$ only acts on $\mathscr{r}$.

First, evaluate $\vec{\nabla}\left(\frac{1}{\mathscr{r}}\right)$:

$$\frac{\partial}{\partial x}\left(\frac{1}{\mathscr{r}}\right) = \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}\right)$$

$$= -\frac{x-x'}{\mathscr{r}^3}$$

$$\frac{\partial}{\partial y}\left(\frac{1}{\mathscr{r}}\right) = -\frac{y-y'}{\mathscr{r}^3}$$

$$\frac{\partial}{\partial z}\left(\frac{1}{\mathscr{r}}\right) = -\frac{z-z'}{\mathscr{r}^3}$$

Then $\vec{\nabla}\left(\frac{1}{\mathscr{r}}\right) = -\frac{1}{\mathscr{r}^3}(x-x', y-y', z-z') = -\frac{\vec{\mathscr{r}}}{\mathscr{r}^3} = -\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \iiint dx'dy'dz' \frac{\mu_0 \vec{J}(\vec{r}\,')}{4\pi \mathscr{r}}$$

$$= \iiint dx'dy'dz' \, \mu_0 \left(\vec{\nabla} \frac{1}{\mathscr{r}}\right) \times \vec{J}(\vec{r}\,') \frac{1}{4\pi}$$

$$= \iiint dx'dy'dz' \, \mu_0 \, -\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \times \frac{\vec{J}(\vec{r}\,')}{4\pi}$$

$$= \iiint dx'dy'dz' \, \frac{\mu_0 \vec{J}(\vec{r}\,') \times \hat{\mathscr{r}}}{4\pi \mathscr{r}^2}$$

Consider a wire with current I:

For a thin wire, we can replace

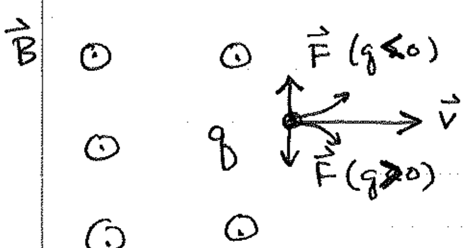
$$\iiint dx'\,dy'\,dz'\ \vec{J} \approx \int \vec{ds}\ I$$

This gives the Biot-Savart law

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{ds} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

Hall effect

Charges moving in a magnetic field experience a force $\vec{F} = q\,\vec{v} \times \vec{B}$



Direction of $\vec{F}$ depends on sign of q.

This effect distinguishes the sign of charged particles, and can be used to tell whether charge carriers in conductors have positive or negative charges.

First, let's expand:

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

Now, plug in to Maxwell's equations:

$$\vec{\nabla} \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} + \frac{\partial^2 A_x}{\partial y \partial z} - \frac{\partial^2 A_z}{\partial y \partial x} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y}$$

$$= 0$$

$$\vec{\nabla} \times \vec{B} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}, \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}, \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right)$$

$$= \left(\frac{\partial}{\partial y}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) - \frac{\partial}{\partial z}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right), \ldots, \ldots \right)$$

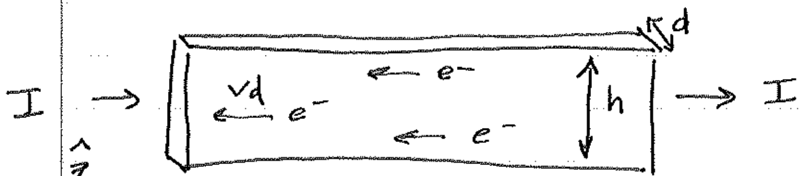
only evaluate x component for simplicity, others are similar

$$= \left(\frac{\partial^2 A_y}{\partial y \partial x} - \frac{\partial^2 A_x}{\partial y^2} - \frac{\partial^2 A_x}{\partial z^2} + \frac{\partial^2 A_z}{\partial z \partial x}, \ldots, \ldots \right)$$

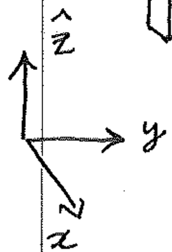
$$= \left(\frac{\partial}{\partial x}\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) - \left(\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \right), \ldots, \ldots \right)$$

add & subtract $\frac{\partial^2 A_x}{\partial x^2}$

Consider current flowing down a strip of height h and thickness d:



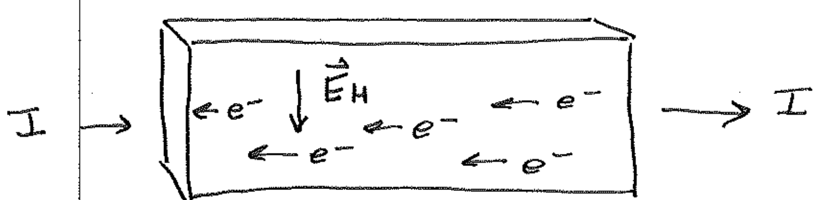
$\vec{v}_d = -v_d \hat{y}$



Now apply external magnetic field $\vec{B} = B\hat{x}$ (out of the page)

$$\vec{F} = -e\vec{v}_d \times \vec{B} = -e v_d B \hat{z}$$

Magnetic force points down. e^- move toward bottom of strip. Causes an electric field $\vec{E}_H$.



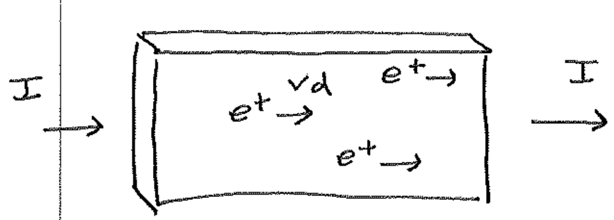
Equilibrium in $\hat{z}$ direction reached when

$$-e\vec{E}_H - e\vec{v}_d \times \vec{B} = 0$$

$$\vec{E}_H = -e v_d B \hat{z} = -\vec{v}_d \times \vec{B}$$

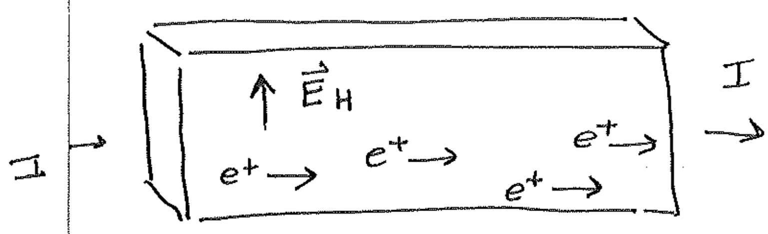
$\Rightarrow$ Potential diffence: $\underset{\Delta\phi}{\underset{\parallel}{\phi(\text{top}) - \phi(\text{bottom})}} = E_H h = v_d B h$

Now imagine if it were the positive charges which moved, while the negative charges were fixed:



Apply same $\vec{B} = B\hat{x}$

$$\vec{F} = e\vec{v}_d \times \vec{B} = -e v_d B \hat{z}$$ Force points down.



Equilibrium in $\hat{z}$ direction: $\vec{E}_H = -\vec{v}_d \times \vec{B} = +v_d B \hat{z}$

Potential difference: $\Delta\phi = \phi(\text{top}) - \phi(\text{bottom}) = -v_d B h$

Can also measure the number density n of charge carriers (free electrons, mostly)

Recall $J = n v_d e$ (here $\vec{J}$ points in $\hat{y}$ direction)
$= I/A = I/dh$

$$\Rightarrow \quad n = \frac{J}{v_d e} = \frac{I}{v_d d h e} = \frac{IB}{\Delta\phi\, d e}$$