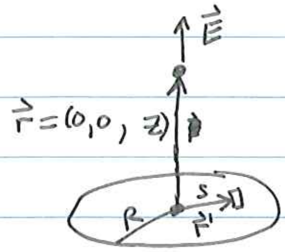
$\vec{E}$ from thin disk of uniform charge σ and radius R



$$\vec{r} = (0,0,z)$$
$$\vec{r}' = (x,y,0) = (s\cos\theta,\ s\sin\theta,\ 0)$$
$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' = (-s\cos\theta,\ -s\sin\theta,\ z)$$
$$\hat{\mathscr{r}} = \frac{1}{\sqrt{s^2+z^2}}(-s\cos\theta,\ -s\sin\theta,\ z)$$

$$dq = s\,ds\,d\theta\ \sigma$$

$$E_z = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathscr{r}^2}\hat{\mathscr{r}}_z$$

$$= \int_0^R s\,ds \int_0^{2\pi} d\theta \frac{1}{4\pi\varepsilon_0}\frac{z\sigma}{(s^2+z^2)^{3/2}}$$

$$= \frac{\sigma z}{2\varepsilon_0}\int_0^R ds \frac{s}{(s^2+z^2)^{3/2}}$$

$$= \frac{\sigma z}{2\varepsilon_0}\cdot \frac{1}{\sqrt{s^2+z^2}}\cancel{(-\tfrac{1}{2})2}\Bigg|_{s=0}^{s=R} = \frac{\sigma z}{2\varepsilon_0}(-1)\left[\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{z}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]\hat{z}$$

limit $R \to \infty$: infinite sheet: $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}$ above sheet.
(note: independent of z)

Solid Hemisphere 

of radius R and uniform charge ρ_0

$\vec{r} = (0,0,0)$

$\vec{r}\,' = (x, y, z) = (r\sin\theta\cos\phi,\ r\sin\theta\sin\phi,\ r\cos\theta)$

$\vec{\mathscr{r}} = -\vec{r}\,'$

only z component contributes

$$E_z = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathscr{r}^2}\mathscr{r}_z = \frac{\rho_0}{4\pi\varepsilon_0}\int_0^R r^2\,dr \int_0^{\pi/2} d\theta \underset{\sin\theta}{\underbrace{1}} \int_0^{2\pi} d\phi\ \frac{1}{r^2}(-\cos\theta)$$

$$= \frac{-\rho_0}{4\pi\varepsilon_0}\underbrace{\int_0^R dr}_{R}\ \underbrace{\int_0^{2\pi} d\phi}_{2\pi}\ \underbrace{\int_0^{\pi/2} d\theta\ \sin\theta\cos\theta}_{\frac{1}{2}\sin^2\theta\,\Big|_0^{\pi/2} = \frac{1}{2}} = -\frac{\rho_0}{4\pi\varepsilon_0}R\,2\pi\,\frac{1}{2} = \frac{-\rho_0 R}{4\varepsilon_0}$$

$$\vec{E} = -\frac{\rho_0 R}{4\varepsilon_0}\hat{z}$$