Tutorial #3 : Line integrals & work

(1) Consider solid sphere with radius R & uniform charge density ρ.

How much work is required to bring a point charge q from ∞ to its center?

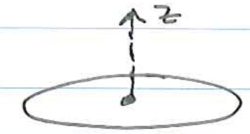
step 1: find $\vec{E}$:
$$\vec{E}(r) = \begin{cases} \frac{\rho r}{3\varepsilon_0}\,\hat{r} & r<R \\ \frac{\rho R^3}{3\varepsilon_0 r^2}\,\hat{r} & r>R \end{cases}$$

step 2: find $\vec{F}$: $\vec{F} = q\vec{E}$

step 3: $\vec{ds} = dr\,\hat{r}$

step 4:
$$W = -\int_{\infty}^{0} \vec{ds}\cdot\vec{F} = -\int_{\infty}^{0} dr\, q\,E(r)$$
$$= -\int_{\infty}^{R} dr\,\frac{q\rho R^3}{3\varepsilon_0 r^2} - \int_{R}^{0} dr\,\frac{q\rho r}{3\varepsilon_0}$$
$$= -\frac{q\rho R^3}{3\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_{\infty}^{R} - \frac{q\rho}{3\varepsilon_0}\frac{r^2}{2}\Big|_{R}^{0}$$
$$= \frac{q\rho}{3\varepsilon_0}\left(R^2 + \frac{R^2}{2}\right) = \frac{q\rho}{2\varepsilon_0}R^2$$

(2) charged disk with radius R and surface charge σ.



Work to move point charge from ∞ to center of disk?

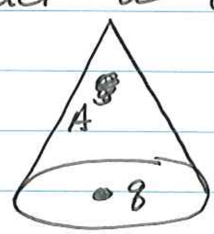
$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{z} \quad \text{for } z>0 \qquad \vec{ds} = dz\,\hat{z}$$

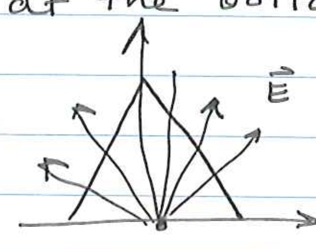
$$W = -\int_{\infty}^{0} dz\, \frac{q\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$= -\frac{q\sigma}{2\varepsilon_0}\left(z - \sqrt{z^2+R^2}\right)\Big|_{\infty}^{0}$$

$$= \frac{q\sigma R}{2\varepsilon_0}$$

(3) Consider a point charge at the origin.
Consider a conical surface over the point charge, which is open at the bottom.





What is the flux through A?

Φ_E is same as hemisphere

$$\Phi_E = \frac{q}{4\pi r^2 \varepsilon_0} 2\pi r^2 = \frac{q}{2\varepsilon_0}$$