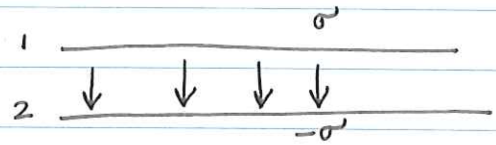
Tutorial #4

example: two infinite, parallel thin plates with surface charge σ and $-\sigma$ respectively, separated by distance d. What is the potential difference between the plates?



$$\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{z}$$

$$\phi_{21} = -\int_0^{-d} dz \left(-\frac{\sigma}{\varepsilon_0}\right) = -\frac{\sigma}{\varepsilon_0} d$$

example: concentric spherical shells with inner radius R_1 and total charge Q and outer radius R_2 and total charge $-Q$. Find potential difference.

surface charges: $\sigma_1 = \frac{Q}{4\pi R_1^2}$, $\sigma_2 = \frac{-Q}{4\pi R_2^2}$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \quad \text{for} \quad R_1 < r < R_2$$

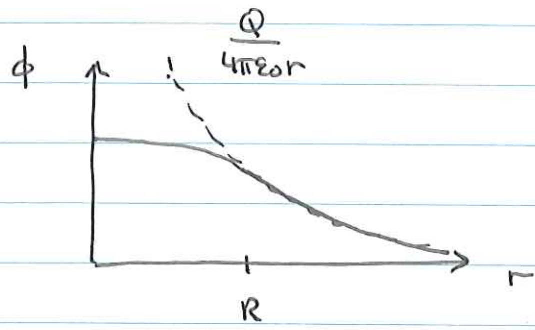
$$\phi_{21} = -\int_{R_1}^{R_2} dr \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

example:

Suppose we have a spherically symmetric potential:

$$\phi(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0 R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right) & r < R \\ \frac{Q}{4\pi\varepsilon_0 r} & r > R \end{cases}$$

Q = total charge of charge configuration.



What is the charge distribution?

For $r > R \to$ same as point charge Q.

For $r < R$:

$$\phi(r) = \frac{Q}{4\pi\varepsilon_0 R}\left(\frac{3}{2} - \frac{x^2+y^2+z^2}{2R^2}\right)$$

$$E_x = -\frac{\partial\phi}{\partial x} = +\frac{Q}{4\pi\varepsilon_0 R}\frac{2x}{2R^2} = \frac{Q}{4\pi\varepsilon_0 R^3}x$$

$$E_y = \frac{Q}{4\pi\varepsilon_0 R^3}y, \quad E_z = \frac{Q}{4\pi\varepsilon_0 R^3}z$$

$$\Rightarrow \vec{E} = \frac{Q}{4\pi\varepsilon_0 R^3}\vec{r}$$

$$E = |\vec{E}| = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

Next, use Gauss's law assuming a spherically symmetric charge distribution.

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3} = \frac{Q_{encl}(r)}{\varepsilon_0 4\pi r^2}$$

$$\Rightarrow Q_{encl}(r) = \frac{Qr^3}{R^3}$$

Now $Q_{encl}(r) = \int dV \, \rho(r')$ $\qquad dV = r^2 dr \sin\theta \, d\theta \, d\phi$

$$= 4\pi \int_0^r dr' \, r'^2 \rho(r')$$

need to use $r \to r'$

$$\frac{dQ_{encl}}{dr} = 4\pi r^2 \rho(r) = \frac{3Q r^2}{R^3}$$

$$\Rightarrow \quad \rho(r) = \frac{Q}{\frac{4\pi}{3} R^3} = \text{const} = \rho$$

Sphere of const. uniform density ρ and radius R.

<u>Potential of Disk</u>

$$\vec{E}(z) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$